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Matrix Depot: an extensible test matrix collection for Julia

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ABSTRACT

Matrix Depot is a Julia software package that provides easy access to a large and diverse collection of test matrices. Its novelty is threefold. First, it is extensible by the user, and so can be adapted to include the user's own test problems. In doing so, it facilitates experimentation and makes it easier to carry out reproducible research. Second, it amalgamates in a single framework two different types of existing matrix collections, comprising parametrized test matrices (including Hansen's set of regularization test problems and Higham's Test Matrix Toolbox) and real-life sparse matrix data (giving access to the University of Florida sparse matrix collection). Third, it fully exploits the Julia language. It uses multiple dispatch to help provide a simple interface and, in particular, to allow matrices to be generated in any of the numeric data types supported by the language.

Subjects Algorithms and Analysis of Algorithms, Data Science, Scientific Computing and Simulation

Keywords Julia, Software package, Test matrices, Matrix algorithm., Test problems

INTRODUCTION

In 1969, Gregory and Karney published a book of test matrices (*Gregory & Karney, 1969*). They stated that "In order to test the accuracy of computer programs for solving numerical problems, one needs numerical examples with known solutions. The aim of this monograph is to provide the reader with suitable examples for testing algorithms for finding the inverses, eigenvalues, and eigenvectors of matrix." At that time it was common for journal papers to be devoted to introducing and analyzing a particular test matrix or class of matrices, examples being the papers of *Clement (1959)* (in the first issue of SIAM Review), *Pei (1962)* (occupying just a quarter of a page), and *Gear (1969)*.

Today, test matrices remain of great interest, but not for the same reasons as fifty years ago. Testing accuracy using problems with known solutions is less common because a reference solution correct to machine precision can usually be computed at higher precision without difficulty. The main uses of test matrices nowadays are for exploring the behavior of mathematical quantities (such as eigenvalue bounds) and for measuring the performance of one or more algorithms with respect to accuracy, stability, convergence rate, speed, or robustness.

Various collections of matrices have been made available in software. As well as giving easy access to matrices these collections have the advantage of facilitating reproducibility of experiments (*Donoho & Stodden, 2015*), whether by the same researcher months later or by different researchers.

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Additional Information and Declarations can be found on page 23

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OPEN ACCESS

¹The University of Florida Sparse Matrix Collection is to be renamed as The SuiteSparse Matrix Collection. An early collection of parametrizable matrices was given by *Higham (1991)* and made available in MATLAB form. The collection was later extended and distributed as a MATLAB toolbox (*Higham, 1995*). Many of the matrices in the toolbox were subsequently incorporated into the MATLAB gallery function. Marques, Vömel, Demmel, and Parlett (*Marques et al., 2008*) present test matrices for tridiagonal eigenvalue problems (already recognized as important by Gregory and Karney, who devoted the last chapter of their book to such matrices). The Harwell–Boeing collection of sparse matrices (*Duff, Grimes & Lewis, 1989*) has been widely used, and is incorporated in the University of Florida Sparse Matrix Collection¹ (*Davis & Hu, 2011*), which contains over 2700 matrices from practical applications, including standard and generalized eigenvalue problems from *Bai et al. (1997)*. Among other MATLAB toolboxes we mention the CONTEST toolbox (*Taylor & Higham, 2009*), which produces adjacency matrices describing random networks, and the NLEVP collection of nonlinear eigenvalue problems (*Betcke et al., 2013*).

The purpose of this work is to provide a test matrix collection for Julia (*Bezanson et al.*, 2014; *Bezanson et al.*, 2012), a new dynamic programming language for technical computing. The collection, called Matrix Depot, exploits Julia's multiple dispatch features to enable all matrices to be accessed by one simple interface. Moreover, Matrix Depot is extensible. Users can add matrices from the University of Florida Sparse Matrix Collection and Matrix Market; they can code new matrix generators and incorporate them into Matrix Depot; and they can define new groups of matrices that give easy access to subsets of matrices. The parametrized matrices can be generated in any appropriate numeric data type, such as

- floating-point types Float16 (half precision: 16 bits), Float32 (single precision: 32 bits), and Float64 (double precision: 64 bits);
- integer types Int32 (signed 32-bit integers), UInt32 (unsigned 32-bit integers), Int64 (signed 64-bit integers), and UInt64 (unsigned 64-bit integers);
- Complex, where the real and imaginary parts are of any Real type (the same for both);
- Rational (ratio of integers); and
- arbitrary precision type BigFloat (with default precision 256 bits), which uses the GNU MPFR Library (*Fousse et al., 2007*).

This paper is organized as follows. We start by giving a brief demonstration of Matrix Depot in 'A Taste of Matrix Depot.' Then we explain the design and implementation of Matrix Depot in 'Package Design and Implementation,' giving details on how multiple dispatch is exploited; how the collection is stored, accessed, and documented; and how it can be extended. In 'The Matrices' we describe the two classes of matrices in Matrix Depot: parametrized test matrices and real-life sparse matrix data. Concluding remarks are given in the final section.

A TASTE OF MATRIX DEPOT

To download Matrix Depot, in a Julia REPL (read-eval-print loop) run the command

```
> Pkg.add("MatrixDepot")
```

Then import Matrix Depot into the local scope.

```
> using MatrixDepot
```

Now the package is ready to be used. First, we find out what matrices are in Matrix Depot.

> mat	rixdepot()							
Matri	ces:							
1)	baart	2)	binom	ial	3)	blur	4)	cauchy
5)	chebspec	6)	chow		7)	circul	8)	clement
9)	companion	10)	deriv	2	11)	dingdong	12)	fiedler
13)	forsythe	14)	foxgo	bd	15)	frank	16)	golub
17)	gravity	18)	grcar		19)	hadamard	20)	hankel
21)	heat	22)	hilb		23)	invhilb	24)	invol
25)	kahan	26)	kms		27)	lehmer	28)	lotkin
29)	magic	30)	minij		31)	moler	32)	neumann
33)	oscillate	34)	paral	lax	35)	parter	36)	pascal
37)	pei	38)	phill	ips	39)	poisson	40)	prolate
41)	randcorr	42)	rando		43)	randsvd	44)	rohess
45)	rosser	46)	sampl	ing	47)	shaw	48)	spikes
49)	toeplitz	50)	tridia	ag	51)	triw	52)	ursell
53)	vand	54)	wathe	n	55)	wilkinson	56)	wing
Group								
all		data		eigen		ill-cond		
inv	erse	pos-def		random		regprob		
spa	rse	symmetr	ic					

All the matrices and groups in the collection are shown. It is also possible to obtain just the list of matrix names.

```
> matrixdepot("all")
56-element Array{ASCIIString,1}:
 "baart"
 "binomial"
 "blur"
 "cauchy"
 "chebspec"
 "chow"
 "circul"
 "clement"
 "companion"
 "deriv2"
  . . .
 "spikes"
 "toeplitz"
 "tridiag"
 "triw"
 "ursell"
 "vand"
 "wathen"
```

```
"wilkinson"
"wing"
```

Here, "..." denotes that we have omitted some of the output in order to save space. Next, we check the input options of the Hilbert matrix hilb.

```
> matrixdepot("hilb")
   Hilbert matrix
   _____
 The Hilbert matrix has (i,j) element 1/(i+j-1). It is notorious
 for being ill conditioned. It is symmetric positive definite
 and totally positive.
 Input options:
   * [type,] dim: the dimension of the matrix.
      [type,] row_dim, col_dim: the row and column dimensions.
 Groups: ["inverse", "ill-cond", "symmetric", "pos-def"]
 References:
 M. D. Choi, Tricks or treats with the Hilbert matrix, Amer. Math.
 Monthly, 90 (1983), pp. 301-312.
 N. J. Higham, Accuracy and Stability of Numerical Algorithms,
 second edition, Society for Industrial and Applied Mathematics,
 Philadelphia, PA, USA, 2002; sec. 28.1.
```

Note that an optional first argument type can be given; it defaults to Float64. The string of equals signs on the third line in the output above is Markdown notation for a header. Julia interprets Markdown within documentation, though as we are using typewriter font for code examples here, we display the uninterpreted source. We generate a 4×6 Hilbert matrix with elements in the default double precision type and then in Rational type.

```
> matrixdepot("hilb", 4, 6)
4x6 Array{Float64,2}:

        1.0
        0.5
        0.333333
        0.25
        0.2

        0.5
        0.333333
        0.25
        0.2
        0.16666

        0.333333
        0.25
        0.2
        0.166667
        0.14285

                                                                    0.166667
                                                        0.166667 0.142857
 0.333333 0.25 0.2
                                          0.166667 0.142857 0.125
                            0.166667 0.142857 0.125
 0.25
               0.2
                                                                      0.111111
> matrixdepot("hilb", Rational, 4, 6)
4x6 Array{Rational{T<:Integer},2}:</pre>
 1//1 1//2 1//3 1//4 1//5 1//6
 1//2 1//3 1//4 1//5 1//6 1//7
 1//3 1//4 1//5 1//6 1//7 1//8
 1//4 1//5 1//6 1//7 1//8 1//9
```

A list of all the symmetric matrices in the collection is readily obtained.

```
> matrixdepot("symmetric")
```

```
21-element Array{ASCIIString,1}:
```

"cauchy"

```
"circul"
"clement"
"dingdong"
"fiedler"
"hankel"
"hilb"
"invhilb"
"kms"
"lehmer"
"minij"
"moler"
"oscillate"
"pascal"
"pei"
"poisson"
"prolate"
"randcorr"
"tridiag"
"wathen"
"wilkinson"
```

Here, symmetric is one of several predefined groups, and multiple groups can be intersected. For example, the for loop below prints the smallest and largest eigenvalues of all the 4×4 matrices in Matrix Depot that are symmetric positive definite and (potentially) ill conditioned.

```
> for name in matrixdepot("symmetric", "pos-def", "ill-cond")
A = full(matrixdepot(name, 4))
@printf "
name eigmin(A) eigmax(A)
end
cauchy: smallest eigval = 2.131e-05, largest eigval = 9.776e-01
hilb: smallest eigval = 9.670e-05, largest eigval = 1.500e+00
invhilb: smallest eigval = 6.666e-01, largest eigval = 1.034e+04
kms: smallest eigval = 3.750e-01, largest eigval = 2.086e+00
moler: smallest eigval = 3.336e-02, largest eigval = 5.122e+00
oscillate: smallest eigval = 1.490e-08, largest eigval = 1.000e+00
pascal: smallest eigval = 3.802e-02, largest eigval = 2.630e+01
pei: smallest eigval = 1.000e+00, largest eigval = 5.000e+00
tridiag: smallest eigval = 3.820e-01, largest eigval = 3.618e+00
```

Matrices can also be accessed by number within the alphabetical list of matrix names.

```
> matrixdepot(2)
"binomial"
> matrixdepot(2:5)
4-element Array{AbstractString,1}:
   "binomial"
   "blur"
   "cauchy"
   "chebspec"
> matrixdepot(15:20, 5, 6, 1:3)
11-element Array{AbstractString,1}:
   "frank"
   "golub"
   "gravity"
   "grcar"
```

```
"hadamard"
"hankel"
"chebspec"
"chow"
"baart"
"binomial"
"blur"
```

Access by number provides a convenient way to run a test on subsets of matrices in the collection. However, the number assigned to a matrix may change if we include new matrices in the collection. In order to run tests in a way that is repeatable in the future it is best to group matrices into subsets using the macro <code>@addgroup</code>, which stores them by name. For example, the following command will group test matrices <code>frank</code>, <code>golub</code>, <code>gravity</code>, <code>grcar</code>, hadamard, hankel, chebspec, chow, baart, binomial, and blur into test1.

> @addgroup test1 = matrixdepot(15:20, 5, 6, 1:3)

After reloading the package, we can run tests on these matrices using group test1. Here we compute the 2-norms. Since blur (an image deblurring test problem) generates a sparse matrix and the matrix 2-norm is currently not implemented for sparse matrices in Julia, we use full to convert the matrix to dense format.

```
> for name in matrixdepot("test1")
        A = full(matrixdepot(name, 4))
        @printf "\%9s has 2-norm \%0.3e \n" name norm(A)
end
        baart has 2-norm 3.192e+00
        binomial has 2-norm 4.576e+00
        blur has 2-norm 8.298e-01
chebspec has 2-norm 6.474e+00
        chow has 2-norm 3.414e+00
        frank has 2-norm 7.624e+00
        golub has 2-norm 7.624e+00
        golub has 2-norm 2.050e+02
        gravity has 2-norm 6.656e+00
        grcar has 2-norm 2.562e+00
hadamard has 2-norm 2.000e+00
        hankel has 2-norm 1.160e+01
```

To download the test matrix SNAP/web-Google from the University of Florida Sparse Matrix Collection (see 'Matrix Data from External Sources' for more details), we first download the data with

```
> matrixdepot("SNAP/web-Google", :get)
```

and then generate the matrix with

```
> matrixdepot("SNAP/web-Google", :r)
916428x916428 sparse matrix with 5105039 Float64 entries:
[11343 , 1] = 1.0
[11928 , 1] = 1.0
[15902 , 1] = 1.0
[29547 , 1] = 1.0
```

[30282 , [31301 ,	1] 1]	=	1.0
[38717 , [720325,	1]	=	1.0
[772226, [785097,	916427] 916427]	=	1.0
[788476, [822938, [833616,	916427] 916427] 916427]	=	1.0 1.0 1.0
[417498, [843845,	916428] 916428] 916428]	=	1.0

Note that the omission marked "..." was in this case automatically done by Julia based on the height of the terminal window. Matrices loaded in this way are inserted into the list of available matrices, and assigned a number. After downloading further matrices HB/1138_bus, HB/494_bus, and Bova/rma10 the list of matrices is as follows.

```
julia> matrixdepot()
Matrices:
  1) baart2) binomial3) blur4) cauchy5) chebspec6) chow7) circul8) clement
 1) baart
 5) chebspec6) chow7) circul9) companion10) deriv211) dingdong13) forsythe14) foxgood15) frank
                                                     12) fiedler
                                                     16) golub
 17) gravity 18) grcar
                                  19) hadamard 20) hankel
 21) heat
                22) hilb
                                  23) invhilb
                                                    24) invol
 25) kahan 26) kms
                                  27) lehmer
                                                     28) lotkin
 29) magic 30) minij 31) moler
                                                     32) neumann
 33) oscillate 34) parallax 35) parter
                                                     36) pascal
 37) pei 38) phillips 39) poisson
                                                     40) prolate
                                 43) randsvd
47) shaw
51) triw
 41) randcorr 42) rando
                                                     44) rohess
                 46) sampling
 45) rosser
                                                     48) spikes

        49) toeplitz
        50) tridiag
        51) triw
        52) urse

        53) vand
        54) wathen
        55) wilkinson
        56) wing

                                                     52) ursell
 57) Bova/rma10 58) HB/1138_bus 59) HB/494_bus 60) SNAP/web-Google
Groups:
                           eigen
 all
                data
                                               ill-cond
               pos-def
                               random
  inverse
                                                 reaprob
  sparse
                 symmetric
                                 test1
```

PACKAGE DESIGN AND IMPLEMENTATION

In this section we describe the design and implementation of Matrix Depot, focusing particularly on the novel aspects of exploitation of multiple dispatch, extensibility of the collection, and user-definable grouping of matrices.

Exploiting multiple dispatch

Matrix Depot makes use of multiple dispatch in Julia, an object-oriented paradigm in which the selection of a function implementation is based on the types of each argument of the function. The generic function matrixdepot has eight different methods, where each method itself is a function that handles a specific case. This is neater and more convenient than writing eight "case" statements, as is necessary in many other languages.

```
> methods(matrixdepot)
# 8 methods for generic function "matrixdepot":
matrixdepot() ...
matrixdepot(name::AbstractString) ...
matrixdepot(name::AbstractString, method::Symbol) ...
matrixdepot(props::AbstractString, args...) ...
matrixdepot(num::Integer) ...
matrixdepot(ur::UnitRange{T<:Real}) ...
matrixdepot(vs::Union{Integer, UnitRange{T<:Real}}...) ...</pre>
```

For example, the following two functions are used for accessing matrices by number and range respectively, where matrix_name_list() returns a list of matrix names. The second function calls the first function in the inner loop.

```
function matrixdepot(num::Integer)
   matrixstrings = matrix_name_list()
   n = length(matrixstrings)
   if num > n
       error("There are $(n) parameterized matrices,
              but you asked for the $(num)-th.")
    end
    return matrixstrings[num]
end
function matrixdepot(ur::UnitRange)
   matrixnamelist = AbstractString[]
    for i in ur
        push!(matrixnamelist, matrixdepot(i))
    end
    return matrixnamelist
end
```

As a result, matrixdepot is a versatile function that can be used for a variety of purposes, including returning matrix information and generating matrices from various input parameters.

In the following example we see how multiple dispatch handles different numbers and types of arguments for the Cauchy matrix.

```
References:
 N. J. Higham, Accuracy and Stability of Numerical Algorithms,
  second edition, Society for Industrial and Applied Mathematics,
 Philadelphia, PA, USA, 2002; sec. 28.1
> matrixdepot("cauchy", [1, 2, 3], [4, 5, 6])
3x3 Array{Float64,2}:
0.2 0.166667 0.142857
0.166667 0.142857 0.125
 0.142857 0.125
                     0.111111
> matrixdepot("cauchy", [0.2, 0.3, 0.4])
3x3 Array{Float64,2}:

        2.5
        2.0
        1.66667

        2.0
        1.66667
        1.42857

1.66667 1.42857 1.25
> matrixdepot("cauchy", 3)
3x3 Array{Float64,2}:
0.5 0.333333 0.25
0.333333 0.25 0.2
0.25 0.2
                    0.166667
> matrixdepot("cauchy", Float32, 3)
3x3 Array{Float32,2}:
0.5 0.333333 0.25
0.333333 0.25 0.2
 0.25
           0.2
                     0.166667
```

Multiple dispatch is also exploited in programming the matrices. For example, the Hilbert matrix is implemented as

```
function hilb{T}(::Type{T}, m::Integer, n::Integer)
    H = zeros(T, m, n)
    for j = 1:n, i = 1:m
        @inbounds H[i,j] = one(T)/ (i + j - one(T))
    end
    return H
end
hilb{T}(::Type{T}, n::Integer) = hilb(T, n, n)
hilb(args...) = hilb(Float64, args...)
```

The function hilb has three methods, which enable one to request, for example, hilb(4,2) for a 4×2 Hilbert matrix of type Float64, or simply (thanks to the final two lines) hilb(4) for a 4×4 Hilbert matrix of type Float64. The keyword @inbounds tells Julia to turn off bounds checking in the following expression, in order to speed up execution. Note that in Julia it is not necessary to vectorize code to achieve good performance (*Bezanson et al., 2014*).

All the matrices in Matrix Depot can be generated using the function call

matrixdepot("matrix_name", p1, p2, ...),

where matrix_name is the name of the test matrix, and p1, p2, ..., are input arguments depending on matrix_name. The help comments for each matrix can be viewed by

calling function matrixdepot ("matrix_name"). We can access the list of matrix names by number, range, or a mixture of numbers and range.

- 1. matrixdepot (i) returns the name of the *i*th matrix;
- 2. matrixdepot (i:j) returns the names of the *i*th to *j*th matrices, where *i* < *j*;
- 3. matrixdepot(i:j, k, m) returns the names of the *i*th, (*i*+1)st, ..., *j*th, *k*th, and *m*th matrices.

Matrix representation

Matrix names in Matrix Depot are represented by Julia strings. For example, the Cauchy matrix is represented by "cauchy". Matrix names and matrix groups are stored as hash tables (Dict). In particular, there is a hash table matrixdict that maps each matrix name to its underlying function and a hash table matrixclass that maps each group to its members.

The majority of parametrized matrices are dense matrices of type Array{T,2}, where T is the element type of the matrix. Variables of the Array type are stored in column-major order. A few matrices are stored as sparse matrices (see also matrixdepot("sparse")), in the Compressed Sparse Column (CSC) format; these include neumann (a singular matrix from the discrete Neumann problem) and poisson (a block tridiagonal matrix from Poisson's equation). Tridiagonal matrices are stored in the built-in Julia type Tridiagonal, which is defined as follows.

```
immutable Tridiagonal{T} <: AbstractMatrix{T}
    dl::Vector{T}  # sub-diagonal
    d::Vector{T}  # diagonal
    du::Vector{T}  # sup-diagonal
    du2::Vector{T}  # supsup-diagonal for pivoting
end</pre>
```

Matrix groups

A group is a subset of matrices in Matrix Depot. There are ten predefined groups, described in Table 1, most of which identify matrices with particular properties. Each group is represented by a string. For example, the group of random matrices is represented by "random". Matrices can be accessed by group names, as was illustrated in 'A Taste of Matrix Depot.'

The macro @addgroup is used to add a new group of matrices to Matrix Depot and the macro @rmgroup removes an added group. All the predefined matrix groups are stored in the hash table matrixclass. The macro addgroup essentially adds a new key-value combination to the hash table usermatrixclass. Using a separate hash table prevents the user from contaminating the predefined matrix groups.

Being able to create groups is a useful feature for reproducible research (*Donoho & Stodden*, 2015). For example, if we have implemented algorithm alg01 and we used circul, minij, and grcar as test matrices for alg01, we could type

```
> @addgroup alg01_group = ["circul", "minij", "grcar"]
```

Table 1Predefined groups.	
Group	Description
all	All the matrices in the collection.
data	The matrix has been downloaded from the University of Florida Sparse Collection or the Matrix Market Collection.
eigen	Part of the eigensystem of the matrix is explicitly known.
ill-cond	The matrix is ill-conditioned for some parameter values.
inverse	The inverse of the matrix is known explicitly.
pos-def	The matrix is positive definite for some parameter values.
random	The matrix has random entries.
regprob	The output is a test problem for regularization methods.
sparse	The matrix is sparse.
symmetric	The matrix is symmetric for some parameter values.

This adds a new group to Matrix Depot (we need to reload the package to see the changes).

julia> matrixdepot()										
Matri	Matrices:									
1)	baart	2)	binomi	al	3)	blur		4)	cauchy	
5)	chebspec	6)	chow		7)	circu	ıl	8)	clement	
9)	companion	10)	deriv2		11)	dingd	long	12)	fiedler	
13)	forsythe	14)	foxgoo	d :	15)	frank	:	16)	golub	
17)	gravity	18)	grcar		19)	hadam	nard	20)	hankel	
21)	heat	22)	hilb		23)	invhi	lb	24)	invol	
25)	kahan	26)	kms		27)	lehme	er	28)	lotkin	
29)	magic	30)	minij		31)	moler		32)	neumann	
33)	oscillate	34)	parall	ax 3	35)	parte	er	36)	pascal	
37)	pei	38)	philli	ps :	39)	poiss	on	40)	prolate	
41)	randcorr	42)	rando	4	43)	rands	vd	44)	rohess	
45)	rosser	46)	sampli	ng ʻ	47)	shaw		48)	spikes	
49)	toeplitz	50)	tridia	g !	51)	triw		52)	ursell	
53)	vand	54)	wathen	1	55)	wilki	nson	56)	wing	
Group										
all		data		eigen			ill-c	and		
	erse	pos-de	f	randor			reqpr			
	rse	symmet		alg01			тедри	00		

We can then run alg01 on the test matrices by

```
> for name in matrixdepot(alg01_group)
    A = matrixdepot(name, n) # n is the dimension of the matrix.
    @printf "Test result for
end
```

Adding new matrix generators

Generators are Julia functions that generate test matrices. When Matrix Depot is first loaded, a directory myMatrixDepot is created. It contains two files, group.jl and generator.jl, where group.jl is used for storing all the user-defined groups (see 'Matrix Group') and generator.jl is used for storing generator declarations. ²Git is a free and open source distributed version control system.

Julia packages are simply Git repositories.² The directory myMatrixDepot is untracked by Git, so any local changes to files in myMatrixDepot do not make the MatrixDepot package "dirty." In particular, all the newly defined groups or matrix generators will not be affected when we upgrade to a new version of Matrix Depot. Matrix Depot automatically loads all Julia files in myMatrixDepot. This feature allows a user to simply drop generator files into myMatrixDepot without worrying about how to link them to Matrix Depot.

A new generator is declared using the syntax include_generator (FunctionName, "fname", f). This adds the new mapping "fname" → f to the hash table matrixdict, which we recall maps each matrix name to its underlying function. Matrix Depot will refer to function f using string "fname" so that we can call function f by matrixdepot ("fname"...). The user is free to define new data types and return values of those types. Moreover, as with any Julia function, multiple values can be returned by listing them after the return statement.

For example, suppose we have the following Julia file rand.jl, which contains two generators randsym and randorth and we want to use them from Matrix Depot. The triple quotes in the file delimit the documentation for the functions.

```
random symmetric matrix
_____
Input options:
* n: the dimension of the matrix
function randsym(n)
 A = zeros(n, n)
 for j = 1:n
     for i = 1:j
         A[i,j] = randn()
         if i != j; A[j,i] = A[i,j] end
      end
 end
  return A
end
....
random orthogonal matrix
Input options:
* n: the dimension of the matrix
.....
randorth(n) = qr(randn(n, n))[1]
```

We can copy the file rand.jl to the directory myMatrixDepot and add the following two lines to generator.jl.

```
include_generator(FunctionName, "randsym", randsym)
include_generator(FunctionName, "randorth", randorth)
```

....

This includes the functions randsym and randorth in Matrix Depot, as we can see by looking at the matrix list (the new entries are numbered 43 and 45).

julia	> matrixdep	pot()						
Matrio	ces:							
1)	baart	2)	binom:	ial	3)	blur	4)	cauchy
5)	chebspec	6)	chow		7)	circul	8)	clement
9)	companion	10)	deriv2	2	11)	dingdong	12)	fiedler
13)	forsythe	14)	foxgo	od	15)	frank	16)	golub
17)	gravity	18)	grcar		19)	hadamard	20)	hankel
21)	heat	22)	hilb		23)	invhilb	24)	invol
25)	kahan	26)	kms		27)	lehmer	28)	lotkin
29)	magic	30)	minij		31)	moler	32)	neumann
33)	oscillate	34)	paral	Lax	35)	parter	36)	pascal
37)	pei	38)	phill:	ips	39)	poisson	40)	prolate
41)	randcorr	42)	rando		43)	randorth	44)	randsvd
45)	randsym	46)	rohes	5	47)	rosser	48)	sampling
49)	shaw	50)	spikes	5	51)	toeplitz	52)	tridiag
53)	triw	54)	ursel	L	55)	vand	56)	wathen
57)	wilkinson	58)	wing					
Groups	s:							
all		data		eigen		ill-con	d	
inve	erse	pos-def		random		regprob		
spai	rse	symmetr	ic					

The new generators can be used just like the built-in ones.

```
> matrixdepot("randsym")
    random symmetric matrix
     _____
   Input options:
     * n: the dimension of the matrix
> matrixdepot("randsym", 4)
4x4 Array{Float64,2}:
-0.00992523 0.174531 -1.73322 -0.765096
0.174531 1.69308 0.269062 0.594058
-1.73322 0.269062 -0.824277 -0.541458
-0.765096 0.594058 -0.541458 -0.480428
> matrixdepot("randorth")
    random orthogonal matrix
    _____
   Input options:
     * n: the dimension of the matrix
> A = matrixdepot("randorth", 4)
4x4 Array{Float64,2}:
-0.233943 0.179893 0.563926 -0.771295
-0.769649 -0.141938 -0.5807 -0.224235
 0.247165 0.832118 -0.449941 -0.20986
-0.540204 0.505046 0.377263 0.557477
> A' *A - eye(4,4)
4x4 Array{Float64,2}:
-2.22045e-16 1.66533e-16 -2.77556e-17 -1.66533e-16
```

```
1.66533e-16-1.11022e-16-3.05311e-161.66533e-16-2.77556e-17-3.05311e-16-1.11022e-161.94289e-16-1.66533e-161.66533e-161.94289e-160.0
```

We can also add group information with the function include_generator. The following lines are put in generator.jl.

```
include_generator(Group, "random", randsym)
include_generator(Group, "random", randorth)
```

This adds the functions randsym and randorth to the group random, as we can see with the following query (after reloading the package).

```
> matrixdepot("random")
10-element Array{ASCIIString,1}:
    "golub"
    "oscillate"
    "randcorr"
    "randorth"
    "randorth"
    "randsym"
    "rohess"
    "rosser"
    "wathen"
```

Documentation

The Matrix Depot documentation is created using the documentation generator Sphinx (http://sphinx-doc.org/) and is hosted at Read the Docs (http://matrixdepotjl.readthedocs. org). Its primary goals are to provide examples of usage of Matrix Depot and to give a brief summary of each matrix in the collection. Matrices are listed alphabetically with hyperlinks to the documentation for each matrix. Most parametrized matrices are presented with heat map plots, which are produced using the Winston package (https://github.com/nolta/Winston.jl), with the color range determined by the smallest and largest entries of the matrix. For example, Fig. 1 shows how the Wathen matrix is documented in Matrix Depot.

THE MATRICES

We now describe the matrices that are provided with, or can be downloaded into, Matrix Depot.

Parametrized matrices

In Matrix Depot v0.5.5, there are 58 parametrized matrices (including the regularization problems described in the next section), most of which originate from the Test Matrix Toolbox (*Higham*, 1995). All these matrices can be generated as matrixdepot("matrix_name", n), where n is the dimension of the matrix.

Many matrices can have more than one input parameter, and multiple dispatch provides a convenient mechanism for taking different actions for different argument types. For

wathen

The Wathen matrix is a sparse, symmetric positive, random matrix arising from the finite element method [wath87]. It is the consistent mass matrix for a regular *nx-by-ny* grid of 8-node elements.





example, the tridiag function generates a tridiagonal matrix from vector arguments giving the subdiagonal, diagonal, and superdiagonal vectors, but a tridiagonal Toeplitz matrix can be obtained by supplying scalar arguments that specify the dimension of the matrix, the subdiagonal, the diagonal, and the superdiagonal. If a single, scalar argument n is supplied then an n-by-n tridiagonal Toeplitz matrix with subdiagonal and superdiagonal -1 and diagonal 2 is constructed. This matrix arises in applying central differences to a second derivative operator, and the inverse and the condition number are known explicitly (*Higham, 2002*, sec. 28.5).

Here is an example of the different usages of tridiag.

```
Input options:
    * [type,] v1, v2, v3: v1 and v3 are vectors of subdiagonal and
     superdiagonal elements, respectively, and v2 is a vector of
     diagonal elements.
    * [type,] dim, x, y, z: dim is the dimension of the matrix, x,
    y, z are scalars. \boldsymbol{x} and z are the subdiagonal and
    superdiagonal elements,
     respectively, and y is the diagonal elements.
    * [type,] dim: x = -1, y = 2, z = -1. This matrix is also
   known as the second difference matrix.
  Groups: ["inverse", "ill-cond", "pos-def", "eigen"]
 References:
  J. Todd, Basic Numerical Mathematics, Vol. 2: Numerical Algebra,
  Birkhauser, Basel, and Academic Press, New York, 1977, p. 155.
> matrixdepot("tridiag", [2,5,6;], ones(4), [3,4,1;])
4x4 Tridiagonal{Float64}:
1.0 3.0 0.0 0.0
2.0 1.0 4.0 0.0
0.0 5.0 1.0 1.0
 0.0 0.0 6.0 1.0
> matrixdepot("tridiag", 4, 5, 3, 1)
4x4 Tridiagonal{Float64}:
3.0 1.0 0.0 0.0
5.0 3.0 1.0 0.0
0.0 5.0 3.0 1.0
0.0 0.0 5.0 3.0
> matrixdepot("tridiag", Int, 4)
4x4 Tridiagonal {Int64}:
 2 -1 0 0
 -1 2 -1 0
 0 -1 2 -1
 0 0 -1 2
```

Test problems for regularization methods

A mathematical problem is ill-posed if the solution is not unique or if an arbitrarily small perturbation of the data can cause an arbitrarily large change in the solution. Regularization methods are an important class of methods for dealing with such problems (*Hansen, 1998; Hansen, 2010*). One means of generating test problems for regularization methods is to discretize a given ill-posed problem.

Matrix Depot contains a group of regularization test problems derived from Hansen's MATLAB Regularization Tools (*Hansen, 1994; Hansen, 2007; Hansen, 2008*) that are mostly discretizations of Fredholm integral equations of the first kind:

$$\int_0^1 K(s,t) f(t) dt = g(s), \quad 0 \le s \le 1.$$

The regularization test problems form the group regprob.

```
> matrixdepot("regprob")
12-element Array{ASCIIString,1}:
    "baart"
    "blur"
    "deriv2"
    "foxgood"
    "gravity"
    "heat"
    "parallax"
    "phillips"
    "shaw"
    "spikes"
    "ursell"
    "wing"
```

Each problem is a linear system Ax = b where the matrix A and vectors x and b are obtained by discretization (using quadrature or the Galerkin method) of K, f, and g. By default, we generate only A, which is an ill-conditioned matrix. The whole test problem will be generated if the parameter matrixonly is set to false, and in this case the output has type RegProb, which is defined as

```
immutable RegProb{T}
A::AbstractMatrix{T} # matrix of interest
b::AbstractVector{T} # right-hand side
x::AbstractVector{T} # the solution to Ax = b
end
```

If r is a generated test problem, then r.A, r.b, and r.x are the matrix A and vectors x and b respectively. If the solution is not provided by the problem, the output is stored as type RegProbNoSolution, which is defined as

```
immutable RegProbNoSolution{T}
A::AbstractMatrix{T} # matrix of interest
b::AbstractVector{T} # right-hand side
end
```

For example, the test problem wing can be generated as follows.

```
> matrixdepot("wing")
    A Problem with a Discontinuous Solution
    -------
Input options:
    * [type,] dim, t1, t2, [matrixonly]: the dimension of matrix
    is dim. t1 and t2 are two real scalars such that 0 < t1 < t2
    < 1. If matrixonly = false, the matrix A and vectors b and x
    in the linear system Ax = b will be generated(matrixonly =
    true by default).
    * [type,] n, [matrixonly]: t1 = 1/3 and t2 = 2/3.
Groups: ["regprob"]</pre>
```

```
References:
  G. M. Wing, A Primer on Integral Equations of the First Kind,
  Society for Industrial and Applied Mathematics, 1991, p. 109.
> A = matrixdepot("wing", 4)
4x4 Array{Float64,2}:
0.031189 0.0921165 0.148804 0.198786
0.0310674 0.0889342 0.134959 0.164156
 0.0309463 0.085862 0.122403 0.13556
 0.0308257 0.0828958 0.111014 0.111945
> r = matrixdepot("wing", 4, false)
Test problems for Regularization Methods
Α:
4x4 Array{Float64,2}:
0.031189 0.0921165 0.148804 0.198786
0.0310674 0.0889342 0.134959 0.164156
 0.0309463 0.085862 0.122403 0.13556
 0.0308257 0.0828958 0.111014 0.111945
b:
4-element Array{Float64,1}:
 0.0804953
0.0751385
 0.0701787
 0.0655842
х:
4-element Array{Float64,1}:
 0.0
 0.5
 0.5
 0.0
> r.x
4-element Array{Float64,1}:
 0.0
 0.5
 0.5
 0.0
```

Matrix data from external sources

Matrix Depot provides access to matrices from Matrix Market (*Boisvert et al., 1997*) and the University of Florida Sparse Matrix Collection (*Davis & Hu, 2011*), both of which contain many matrices taken from applications. In particular, these sources contain many large, sparse matrices.

Matrix Market and the University of Florida Sparse Matrix Collection both categorize matrices by application domain and the problem source and both provide matrices in Matrix Market Format (*Boisvert, Pozo & Remington, 1996*). These similarities allow us to design a generic interface for both collections. The symbol :get (or :g) is used for downloading matrices from both collections and the symbol :read (or :r) is used for reading in matrices already downloaded. Downloaded matrix data is stored on disk in the Matrix Market format and when read into Julia is stored in the type SparseMatrixCSC.

MatrixDepot.update() downloads the matrix name data files from the two web servers.

> MatrixDe	pot.update()			
\% Total	\% Received \% >	Xferd Average	Speed Time	Time Time Current
		Dload	Upload Total	Spent Left Speed
100 1887k	0 1887k 0	0 97337	0::	0:00:19: 472k
\% Total	\% Received \% >	Xferd Average	Speed Time	Time Time Current
		Dload	Upload Total	Spent Left Speed
100 41552	0 41552 0	0 4421	0::	0:00:09: 41018

The University of Florida Sparse Matrix Collection is divided into matrix groups and the group of a matrix forms part of the full name of the matrix (*Davis & Hu*, 2011). For example, the full name of the matrix 1138_bus in the Harwell-Boeing Collection is HB/1138_bus.

\% Total	\% Recei	ved \% Xf	erd	Average	e Speed	Time	Ti	.me	Time	Current
				Dload	Upload	Tota	l Sp	ent	Left	Speed
100 19829	100 19829	0	0	2320	0	0:00:0	8 0:0	0:08 -	::-	- 49572
matrixdepo	ot("HB/1138	_bus", :r	ead)							
138x1138 Sy	mmetric{Fl	oat64,Spa	rseM	atrixCS	C{Float(54 , Int6	4}}:			
1474.78	0.0	0.0 .	••	0.0	0.0		0.0	0.0)	
0.0	9.13665	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	69.6147		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
-9.01713	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0 .		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	-3.40599	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0 .		0.0	-24.39	902	0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	0.0)	
0.0	0.0	0.0		26.5639	0.0		0.0	0.0)	
0.0	0.0	0.0 .	•••	0.0	46.1	767	0.0	0.0)	
0.0	0.0	0.0		0.0	0.0	10	000.0	0.0)	
0.0	0.0	0.0		0.0	0.0		0.0	117.6	547	

Matrices from the University of Florida Sparse Matrix Collection are stored in MatrixDepot/data/uf and they are stored by group (to avoid duplicate names), i.e., one directory per group. Similarly, matrices from Matrix Market are stored in MatrixDepot/data/mm. Both directories are untracked by Git. Many matrices in the University of Florida Sparse Matrix Collection contain problem-specific metadata, all of which is downloaded. The metadata is accessed by setting the keyword argument meta to true. Then instead of returning the matrix, Matrix Depot will return the metadata (including the matrix) as a dictionary. For example, the IMDB movie database Pajek/IMDB has metadata related to actors and movies. The following command stores all the metadata of Pajek/IMDB in a variable r, where r["IMDB"] is the matrix.

```
> r = matrixdepot("Pajek/IMDB", :r, meta = true)
Dict{AbstractString,Any} with 8 entries:
   "IMDB_colname" => "/La Tata' Castro, Maria Tereza\n'La Veneno'...
   "IMDB_MovieBacon" => 428440x1 Array{Float64,2}
   "IMDB_code" => "Drama\nShort\nDocumentary\nComedy\nWestern\nFamily...
   "IMDB_KevinBacon" => 1x1 Array{Float64,2}
   "IMDB_ActorBacon" => 896308x1 Array{Float64,2}
   "IMDB_category" => 428440x1 Array{Float64,2}
   "IMDB" => 428440x1 Array{Float64,2}
   "IMDB" => 428440x896308 sparse matrix with 3782463 Float64 entries
   "IMDB_year" => 428440x1 Array{Float64,2}
```

We can download a whole group of matrices from the University of Florida sparse matrix collection using the command matrixdepot ("group name/*", :get). The next example downloads all 67 matrices in the Gset group of matrices from random graphs (contributed by Y. Ye) then displays all the matrices in Matrix Depot, including the newly downloaded matrices.

```
> matrixdepot("Gset/*", :get)
Downloading all matrices in group Gset ...
 \% Total \% Received \% Xferd Average Speed Time Time Time Current
                       Dload Upload Total Spent Left Speed
100 48083 100 48083 0 0 95388 0 ------- ------- 96166
download:/home/weijian/.julia/v0.4/MatrixDepot/src/../data/uf/Gset/G1.tar.gz
G1/G1.mtx
 \% Total
          \% Received \% Xferd Average Speed Time Time
                                                     Time Current
                         Dload Upload Total Spent Left Speed
100 55180 100 55180 0 0 75318 0 --:--:- --:-- 75692
download:/home/weijian/.julia/v0.4/MatrixDepot/src/../data/uf/Gset/G10.tar.gz
G10/G10 mt x
\% Total \% Received \% Xferd Average Speed Time Time Time Current
                       Dload Upload Total Spent Left Speed
100 5926 100 5926 0 0 23126 0 --:--:- --:-- 23515
download:/home/weijian/.julia/v0.4/MatrixDepot/src/../data/uf/Gset/G11.tar.gz
G11/G11 mt x
\% Total \% Received \% Xferd Average Speed Time Time Time Current
              Dload Upload Total Spent Left Speed
100 6349 100 6349 0 0 24223 0 --:--:- --:-- 24608
. . .
> matrixdepot()
Matrices:
 1) baart
                 2) binomial
                                 3) blur
                                                 4) cauchy
 5) chebspec
                 6) chow
                                 7) circul
                                                 clement
 9) companion
                10) deriv2
                                11) dingdong
                                                12) fiedler
                                                16) golub
 13) forsythe
                14) foxgood
                                15) frank
                18) grcar
                                                20) hankel
                                19) hadamard
 17) gravity
                22) hilb
                                23) invhilb
                                                24) invol
 21) heat
                26) kms
                                27) lehmer
                                                28) lotkin
 25) kahan
 29) magic 30) minij
33) oscillate 34) parallax
                30) minij
                                31) moler
                                                 32) neumann
                                35) parter
                                                36) pascal
```

	37)	pei	38)	phillips	39)	poisson	40)	prolate
	41)	randcorr	42)	rando	43)	randsvd	44)	rohess
	45)	rosser	46)	sampling	47)	shaw	48)	spikes
	49)	toeplitz	50)	tridiag	51)	triw	52)	ursell
	53)	vand	54)	wathen	55)	wilkinson	56)	wing
	57)	Gset/G1	58)	Gset/G10	59)	Gset/G11	60)	Gset/G12
	61)	Gset/G13	62)	Gset/G14	63)	Gset/G15	64)	Gset/G16
	65)	Gset/G17	66)	Gset/G18	67)	Gset/G19	68)	Gset/G2
	69)	Gset/G20	70)	Gset/G21	71)	Gset/G22	72)	Gset/G23
ĺ	73)	Gset/G24	74)	Gset/G25	75)	Gset/G26	76)	Gset/G27
	77)	Gset/G28	78)	Gset/G29	79)	Gset/G3	80)	Gset/G30
	81)	Gset/G31	82)	Gset/G32	83)	Gset/G33	84)	Gset/G34
	85)	Gset/G35	86)	Gset/G36	87)	Gset/G37	88)	Gset/G38
	89)	Gset/G39	90)	Gset/G4	91)	Gset/G40	92)	Gset/G41
	93)	Gset/G42	94)	Gset/G43	95)	Gset/G44	96)	Gset/G45
	97)	Gset/G46	98)	Gset/G47	99)	Gset/G48	100)	Gset/G49
	101)	Gset/G5	102)	Gset/G50	103)	Gset/G51	104)	Gset/G52
	105)	Gset/G53	106)	Gset/G54	107)	Gset/G55	108)	Gset/G56
	109)	Gset/G57	110)	Gset/G58	111)	Gset/G59	112)	Gset/G6
	113)	Gset/G60	114)	Gset/G61	115)	Gset/G62	116)	Gset/G63
	117)	Gset/G64	118)	Gset/G65	119)	Gset/G66	120)	Gset/G67
	121)	Gset/G7	122)	Gset/G8	123)	Gset/G9		
	Groups	s:						
	all		data	eigen	i	ll-cond		
	inve	erse	pos-def	random	r	egprob		
	spai	rse	symmetric					
1								

The full name of a matrix in Matrix Market comprises three parts: the collection name, the set name, and the matrix name. For example, the full name of the matrix BCSSTK14 in the set BCSSTRUC2 from the Harwell-Boeing Collection is Harwell-Boeing/bcsstruc2/bcsstk14. Note that both set name and matrix name are in lower case.

1	larwell-Boeing/bcssti		
\% Total \%	Received \% Xferd	Average Speed Ti	me Time Time Current
	Dlo	ad Upload Total	Spent Left Speed
100 292k 100	292k 0 0 226	35 0 0:00:13	3 0:00:13: 61144
download:/home/w	veijian/.julia/v0.4/N	MatrixDepot/data/mm	1/Harwell-Boeing/bcsstruc2
/bcsstk14.mtx.gz			
> matrixdepot("H	Marwell-Boeing/bcsst:	uc2/bcsstk14", :re	ead)
-	ric{Float64,SparseMa		
1.93161e6	. , .	. ,	0.0
0.0	1.0 0.0	0.0	0.0
-1.02166e5			0.0
-35568.9	0.0 1.65787e5		0.0
			0.0
-1.06959e5			
	0.0 35568.9	0.0	0.0
-717.845	0.0 0.0	0.0	0.0
0.0	0.0 88998.5	0.0	0.0
0.0	0.0 -1.82865e6	0.0	0.0
0.0	0.0 1.24988e5	0.0	0.0
0.0	0.0 0.0	0.0	0.0
0.0	0.0 0.0	1.06103e	-5.25151e5
0.0	0.0 0.0	-5.25151e	-53434.0

0.0	0.0	0.0		1.06959e5	-1.65835e5
0.0	0.0	0.0		0.0	0.0
0.0	0.0	0.0		1.06959e5	35568.9
0.0	0.0	0.0	-8165	18.0	1.21311e7
0.0	0.0	0.0		4.55624e7	8.15266e5
0.0	0.0	0.0		8.15266e5	5.27942e8

We recommend downloading matrices from the University of Florida Sparse Matrix Collection when there is a choice, because almost every matrix from Matrix Market is included in it.

CONCLUDING REMARKS

Matrix Depot follows in the footsteps of earlier collections of matrices. Its novelty is threefold. First, it is extensible by the user, and so can be adapted to the user's needs. In doing so it facilitates experimentation, and in particular makes it easier to do reproducible research. Second, it combines several existing test matrix collections, namely Higham's Test Matrix Toolbox, Hansen's regularization problems, and the University of Florida Sparse Matrix Collection, in order to provide both parametrized test matrices and real-life sparse matrix data in a single framework. Third, it fully exploits the Julia language. It uses multiple dispatch to help provide a simple interface and, in particular, to allow matrices to be generated in any of the numeric data types supported by the language. Matrix Depot therefore anticipates the development of intrinsic support in Julia for computations with BigFloat and other data types.

Matrix Depot has been in development since 2014. It is an open source project (https://github.com/weijianzhang/MatrixDepot.jl) hosted on GitHub and is available under the MIT License. A first release was announced in December 2014. Matrix Depot v0.5.5 is the latest official release and consists of around 3,000 lines of source code, with test coverage of 98.91% according to Codecov (https://codecov.io/). From GitHub traffic analytics, we learn that Matrix Depot has 40–70 unique downloads (unique cloners) every month. Matrix Depot also benefits the development of other Julia packages. LightGraphs (https://github.com/JuliaGraphs/LightGraphs.jl), an optimized graph package for Julia, for example, has embedded Matrix Depot as its database.

We built Matrix Depot to facilitate the development and testing of matrix (and other) algorithms in Julia. and we will continue to develop Matrix Depot by introducing new test matrices and integrating other test collections.

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ADDITIONAL INFORMATION AND DECLARATIONS

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Competing Interests

Nicholas J. Higham is an Academic Edtor for PeerJ Computer Science.

Author Contributions

- Weijian Zhang conceived and designed the experiments, performed the experiments, analyzed the data, contributed reagents/materials/analysis tools, wrote the paper, prepared figures and/or tables, performed the computation work, reviewed drafts of the paper.
- Nicholas J. Higham conceived and designed the experiments, analyzed the data, contributed reagents/materials/analysis tools, wrote the paper, performed the computation work, reviewed drafts of the paper.

Data Availability

The following information was supplied regarding data availability: Matrix Depot: https://github.com/weijianzhang/MatrixDepot.jl.

REFERENCES

- **Bai Z, Day D, Demmel J, Dongarra J. 1997.** A test matrix collection for non-Hermitian eigenvalue problems (release 1.0). Technical Report CS-97-355, Department of Computer Science. Knoxville: University of Tennessee, Knoxville. LAPACK Working Note 123, pp. 45.
- Betcke T, Higham NJ, Mehrmann V, Schröder C, Tisseur F. 2013. NLEVP: a collection of nonlinear eigenvalue problems. ACM Transactions on Mathematical Software 39(2): 7:1–7:28.
- **Bezanson J, Edelman A, Karpinski S, Shah VB. 2014.** Julia: a fresh approach to numerical computing. ArXiv preprint. arXiv:1411.1607.
- **Bezanson J, Karpinski S, Shah VB, Edelman A. 2012.** Julia: a fast dynamic language for technical computing. ArXiv preprint. arXiv:1209.5145.
- **Boisvert RF, Pozo R, Remington K, Barrett RF, Dongarra JJ. 1997.** Matrix Market: a Web resource for test matrix collections. In: Boisvert RF, ed. *Quality of numerical software: assessment and enhancement*. London: Chapman and Hall, 125–136.

- **Boisvert RF, Pozo R, Remington KA. 1996.** The Matrix Market exchange formats: Initial design. Technical Report NISTIR 5935. Gaithersburg: National Institute of Standards and Technology, pp. 14.
- Clement PA. 1959. A class of triple-diagonal matrices for test purposes. *SIAM Review* 1(1):50–52 DOI 10.1137/1001006.
- **Davis TA, Hu Y. 2011.** The University of Florida Sparse Matrix Collection. *ACM Transactions on Mathematical Software* **38**(1): 1:1–1:25. *Available at http://www.cise. ufl.edu/research/sparse/matrices.*
- **Donoho DL, Stodden V. 2015.** Reproducible research in the mathematical sciences. In: Higham NJ, Dennis MR, Glendinning P, Martin PA, Santosa F, Tanner J, eds. *The Princeton companion to applied mathematics*. Princeton: Princeton University Press, 916–925.
- **Duff IS, Grimes RG, Lewis JG. 1989.** Sparse matrix test problems. *ACM Transactions on Mathematical Software* **15**(1):1–14 DOI 10.1145/62038.62043.
- **Fousse L, Hanrot G, Lefèvre V, Pélissier P, Zimmermann P. 2007.** MPFR: a multipleprecision binary floating-point library with correct rounding. *ACM Transactions on Mathematical Software* **33(2)**: 13:1–13:15.
- **Gear CW. 1969.** A simple set of test matrices for eigenvalue programs. *Math. Comp.* **23(105)**:119–125 DOI 10.1090/S0025-5718-1969-0238477-8.
- **Gregory RT, Karney DL. 1969.** *A collection of matrices for testing computational algorithms.* New York: Wiley. Reprinted with corrections by Robert E. Krieger, Huntington, New York, 1978.
- Hansen PC. 1994. Regularization Tools: a Matlab package for analysis and solution of discrete ill-posed problems. *Numerical Algorithms* 6(1):1–35DOI 10.1007/BF02149761.
- **Hansen PC. 1998.** *Rank-deficient and discrete ill-posed problems: numerical aspects of linear inversion.* Philadelphia: Society for Industrial and Applied Mathematics, xvi+247 pp.
- Hansen PC. 2007. Regularization Tools version 4.0 for Matlab 7.3. *Numer. Algorithms* 46(2):189–194 DOI 10.1007/s11075-007-9136-9.
- Hansen PC. 2008. Regularization tools. A Matlab package for analysis and solution of discrete ill-posed problems. Version 4.1 for Matlab 7.3. Report, Information and Mathematics Modelling. Technical University of Denmark, DK-2800 Lyngby, Denmark.
- Hansen PC. 2010. *Discrete inverse problems: insight and algorithms*. Philadelphia: Society for Industrial and Applied Mathematics, xii+213 pp.
- Higham NJ. 1991. Algorithm 694: a collection of test matrices in MATLAB. *ACM Transactions on Mathematical Software* 17(3):289–305 DOI 10.1145/114697.116805.
- **Higham NJ. 1995.** The Test Matrix Toolbox for MATLAB (version 3.0). Numerical Analysis Report No. 276, Manchester Centre for Computational Mathematics. Manchester, England.
- **Higham NJ. 2002.** *Accuracy and stability of numerical algorithms*. second edition. Philadelphia: Society for Industrial and Applied Mathematics, xxx+680 pp.

- Marques OA, Vömel C, Demmel JW, Parlett BN. 2008. Algorithm 880: a testing infrastructure for symmetric tridiagonal eigensolvers. *ACM Transactions on Mathematical Software* 35(1): 8:1–8:13.
- Pei ML. 1962. A test matrix for inversion procedures. *Communications of the ACM* 5(10):508 DOI 10.1145/1462173.1462175.
- **Taylor A, Higham DJ. 2009.** CONTEST: a controllable test matrix toolbox for MATLAB. *ACM Transactions on Mathematical Software* **35**(**4**): 26:1–26:17.