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1 Introduction

I started this article as a set of notes for a talk that I gave to the Numerical Linear Algebra Group, in the School of Mathematics at the University of Manchester, to commemorate the 30th anniversary of the death of James Hardy Wilkinson, or Jim Wilkinson as he was affectionately known for much of his working life. The anniversary was on The 5th of October 2016.

Jim Wilkinson had a big influence on me and so, before it was too late, I felt that I needed to expand on the notes and to give a personal perspective on Wilkinson, and hope that it might also be of interest to others. This task has been seriously delayed, but the occasion of the workshop on Advances in Numerical Linear Algebra: Celebrating the Centenary of the Birth of James H. Wilkinson, May 29–30, 2019 has prompted me back into action.

Jim was originally known as Wilkie, but I always knew him as Jim. Because some of the article concerns my personal relationship with Wilkinson, it is perhaps helpful to give some context. Directly following my graduation, at the slightly late age of 26, with a B.Sc. at Enfield College of Technology in 1968, I became a lecturer, then senior lecturer, in numerical analysis on the same degree course for eleven years¹. Whilst I was there, the college became Middlesex Polytechnic and is now Middlesex University. In 1979 I moved to the National Physical Laboratory (NPL) on a three year research contract before joining the Numerical Algorithms Group (NAG) in Oxford, where I worked for about thirty years and, at the time of writing, still hold an honorary position.

I want to thank Nick Higham and Sam Relton for encouraging me to write up my notes,

2 Contributions

In my view Wilkinson's main technical contributions concern:

- Pilot ACE
- Backward error analysis
- Analysis of algorithms for numerical linear algebra
- Software implementing the algorithms

He also gave strong encouragement to people and projects, particularly software projects such as Eispack, LINPACK and NAG.

I shall elaborate on each of these items as the article proceeds.

 $^{^{1}\}mathrm{I}$ feel so lucky to have had the opportunity to become a lecturer straight after graduating, an opportunity that is not available to graduates today.

3 Career

A brief summary of Wilkinson's career is:

- 1936–1939: Trinity College, Cambridge. Twice most distinguished student in any subject.
- WWII: Ministry of Supply (ballistics, thermodynamics of explosions, ...).
- 1946–1980: National Physical Laboratory. Promoted to Chief Scientific Officer, 1974.

Wilkinson's tutors at Cambridge included G. H. Hardy, J. E. Littlewood and A. S. Besicovitch, and in 1939 at the age of 19 he graduated with a BA in mathematics, with first class honours and distinction, His studying was curtailed when he was drafted into military scientific work in January 1940, initially with the Ordnance Board at Cambridge, but in mid-1943 he was transferred to the Armament Research Laboratory at Fort Halstead. At Cambridge Wilkinson's main interest had been classical analysis, but with the Ministry of Supply he got his first taste of numerical analysis.

Jim also met his wife to be, Heather Norah Ware², at Fort Halstead, where she had been posted after herself obtaining a first class honours degree in mathematics from King's College, London in 1943. With her knowledge of mathematics, Heather became a great help to Jim in preparing papers and lecture notes with him. Wilkinson also met E. T. Goodwin, who subsequently offered Wilkinson a position at NPL.

Cambridge later made Wilkinson a D.Sc. based upon his scientific contributions.

Wilkinson's promotion to Chief Scientific Officer was a special merit position and very rare.

4 Selected Awards

1962 D.Sc. from the Cambridge University

1969 Fellow of the Royal Society

1970 ACM Turing Award and SIAM von Neumann Award

- 1973 Distinguished Fellow of the British Computing Society
- 1974 Foreign Honorary Member of the American Academy of Arts and Sciences
- 1977 Honorary Fellow of the Institute of Mathematics and its Applications
- 1983 Fellow of the Japan Society for the Promotion of Science
- **1987** Chauvenet Prize of the Mathematical Association of America (for his paper on The Perfidious Polynomial, Wilkinson [33])

 $^{^{2}3}$ January, 1924 – 4 February, 2011

At the time of writing, it is still unique to have received both the ACM Turing Award and the von Neumann Award in the same $year^3$

Wilkinson also received a number of honorary degrees and gave many distinguished lectures. Argonne National Laboratory (ANL) in the US, where he visited many times, has a J. H. Wilkinson Fellowship in Computational Science. SIAM has two Prizes in his honour, the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing, and the James H. Wilkinson Prize for Numerical Software. The latter prize was originally initiated by ANL, NAG and NPL, but is now administered by SIAM.

Wilkinson was a frequent visitor to Stanford University, where he gave lecture courses, which were much appreciated by the attendees⁴.

5 The Person

In summary:

- Lifelong marriage to Heather
- Two children, Jenny and David
- Warm caring person with a lovely sense of humour
- Liked food, drink and music
- Practical

Jenny very sadly died of cancer at age 26 in 1979.

On the practical side, Jim did most of the decorating at home himself, although Heather complained that he never got rid of anything! They also had a camper van which they would often use to go on holidays to France, with Jenny and David, and Jim did much of the servicing on the van.

Jim also loved classical music and had an encyclopaedic knowledge of the music.

6 Personal Reflection

I want to briefly give some of my personal interactions with Wilkinson in order to illustrate just what a kind and thoughtful person he was.

Towards the end of the degree course that I took, we had to do a mini thesis and mine was on Latent Roots and Latent Vectors⁵. One of the two external examiners was from the

³The lectures were published as [29, 1971] and [30, 1971].

⁴Steve Leon had kept Wilkinson's notes for two of his courses and very kindly had them copied to pdf files. These are available at: https://nla-group.org/james-hardy-wilkinson/.

⁵Of course, we now commonly call them eigenvalues and eigenvectors! Although Wilkinson did himself publish three articles with latent roots in their title!!

NPL and seemed to be impressed by the thesis, so he kindly invited me to NPL to meet Wilkinson. During the visit Wilkinson told me about some work on plane rotations which, with his encouragement, led to my first published paper, in 1974^6 . In 1975/76 I went to NPL for a sabbatical year with Wilkinson and we wrote two technical reports together, one on the practical behaviour of SOR, of which I am still quite proud, [11]. I am not sure that I realised at the time what an honour it was, but the experience changed the course of my career.

At that time the rules of Wilkinson's post required him to retire at sixty. In 1979, shortly before his retirement, I was offered a three year research contract looking after Wilkinson's small group⁷. He lived close by and would come to my office regularly to talk, especially about his daughter Jenny. I felt so privileged that he would trust me with his feelings. I just hope that it helped him, at least a little. I also learned about his work with Alan Turing and about his conversion to backward error analysis.

Of course, Wilkinson knew that I was never going to be a top rank numerical analyst, but that did not affect his encouragement. He was a wonderful friend. Although Wilkinson never had a full time academic position, it did not prevent him from encouraging many others. He was also very appreciative of the people who worked directly with him at NPL. Wilkinson gave Gwen Peters a copy of the Algebraic Eigenvalue Problem and she recently showed me the dedication that Jim had written in the front, see Figure 1. She says that she was privileged to have worked with Wilkinson and that he always saw the best in people, and that was my experience also.

To Gaven, in appreciation of the contribution to The practical rectization of them results

Figure 1: Dedication to Gwen Peters

Despite 40 Atbara Road not being a large house, Heather and Jim hosted many large dinner parties. They were wine connoisseurs and much of their collection was stored under the furniture! It was at one of Heather and Jim's dinner parties that Brian Ford, then Director of NAG, kindly invited me to apply for a job when my contract at NPL ended. It was

⁶A short note on fast plane rotations, nowadays of no great significance, [9]. It took me a further eight years to publish my second paper!

 $^{^{7}}$ Wilkinson officially retired in January 1980, but I believe that he was at Stanford University for the latter part of 1979.

not until some considerable time after I had joined NAG that I learned that Wilkinson had written in strong support of my application; a mark of his caring attitude that he would quietly do that without being asked.

7 Turing and Pilot ACE

Jim Wilkinson's career was inextricably linked to Alan Turing. Wilkinson had thought about going back to Cambridge to study classical analysis. But, after working with Turing, he said with his typical humour "Had it not been for Turing I would probably have been just a pure mathematician", apparently said with a suitably pejorative flavour! See Wilkinson [29, 1971] for some comments on working with Turing, and for examples of Wilkinson's humour.

Turing's time at Bletchley Park and his code breaking work is very well documented, so I shall not describe it here. Suffice it to say that his work at Bletchley Park and his famous computing paper [16, 1937] put him in an almost unique position to design a computer. In 1945 Turing was employed by the Department of Scientific and Industrial Research at NPL to do just that, to design and then build an Automatic Computing Engine (ACE).

In 1946 Wilkinson also joined NPL, nominally half time with Turing and half time with the Desk Machine Computing Section, using Brunsvigas to solve numerical and statistical problems⁸. Wilkinson said that he did not know whether Turing was undecided if he required Wilkinson's services at all, or if Wilkinson would be so effective that half time would be adequate! [32, 1980, page 102]. In practice, Wilkinson spent most of his time with Turing.

Turing became very unhappy with decisions being made by management about the future of ACE and the resources being assigned to the project, so that, after spending a sabbatical year at Cambridge in 1947, he left for the University of Manchester in 1948.

Just to add a bit about Turing, he was well known as a runner and Wilkinson told me about Turing running to external meetings, somewhat to the surprise of the other attendees. He was a member of the Walton Athletic Club and was at one time the club champion at three miles, and in the ten mile road race. He competed in the marathon in the AAA championships, coming fifth, and may well have been selected for the 1948 London Olympics had he not injured a leg.⁹

Wilkinson himself enjoyed cycling and it was not unknown for him to arrive at meetings on his bicycle.

 $^{^{8}}$ They also solved systems of linear equations using a modified Hollerith machine, see Fox, Huskey, and Wilkinson [7, 6, 1948].

⁹The School of Mathematics at the University of Manchester is located in the The Alan Turing building, and has on display a lovely photo of Alan Turing competing in the 1946 Boxing Day three mile handicap race. The photo can also be seen in Turing [17, 2015, page 207].

8 Pilot ACE

Turing had envisaged that ACE would use 200 long delay lines with some 6,000 words. This was felt to be too ambitious and, following a visit by Harry Huskey¹⁰, work started on a less ambitious project, Pilot ACE, with mercury delay lines with 128 words of 32 binary digits each, later expanded to 352 words. It used a technique called optimum coding¹¹, a design unique to Turing, which meant that programmed floating point arithmetic, via subroutines, was almost as fast as fixed point arithmetic.

Once Turing had left, Wilkinson was effectively in charge of the Pilot ACE project. The computer first worked in 1950 and in November 1950 a successful three day press demonstration was held. Pilot ACE and Wilkinson even made it into a Daily Mirror cartoon! See Copeland [2, 2005, page 103]¹²

Wilkinson said that "Once the machine was a success, however, there were no sour grapes from Turing and he was always extremely generous about what had been achieved." [29, 1971, page 141].

Wilkinson wrote three reports on the design of Pilot ACE and its programming¹³, and in 1954 wrote a substantial article on solving eigenvalue problems on Pilot ACE, [25, 1954]. They were able to solve problems of orders up to sixty, coming from applications such as the aircraft flutter problem¹⁴, quite an achievement given the capacity of Pilot ACE. They routinely computed the residuals $Ax - \lambda x$ for the computed roots and vectors as a check on their accuracy, see page 558, Part IV of the above reference. This experience led Wilkinson to his development of backward error analysis, see: https://nla-group.org/2019/02/18/wilkinson-and-backward-error-analysis/.

Another member of the Pilot ACE team was Donald Davies who, in 1965, developed the idea of packet switching, an essential part of the development of the internet¹⁵. Gwen Peters, whom I mentioned in Section 6, joined the Maths Division at NPL in 1949 aged nineteen and initially performed desk computing in an office next to Leslie Fox¹⁶. When Pilot ACE became operational, she was one of the operators and remembers that they would feed in the punched cards and watch the console lights to see the operations proceed. To quote from Wilkinson, [32, 1980, page 112]:

¹⁵Paul Baran in the US independently developed similar ideas.

¹⁰https://en.wikipedia.org/wiki/Harry_Huskey

¹¹Wilkinson [26, 1955]

 $^{^{12}}$ A Google search for "the pilot ace at the national physical laboratory daily mirror cartoon" may yield a link to the cartoon.

 $^{^{13}}$ [22, 1948], [23, 1951], [24, 1952]. See also [31, 1975].

¹⁴Olga Taussky-Todd worked on flutter problems during the war at NPL. She and her husband, Jack Todd, left for the US in 1946, but if my memory serves me correctly, Wilkinson talked to me about meeting Olga at NPL before she left and discussing the flutter problem. Perhaps she stimulated his interest in eigenvalue problems?

¹⁶Leslie Fox was appointed Professor of Numerical Analysis at Oxford University in 1957, and established the Oxford University Computing Laboratory. In 2017 a Blue Plaque honouring Leslie Fox was unveiled at Dewsbury railway station, alongside one for Tom Kilburn, one of the developers of the "Manchester Baby" computer. They had both attended the Wheelwright Grammar School in Dewsbury, https://www.examinerlive.co.uk/news/blue-plaques-unveiled-recognition-two-13361771.



Figure 2: Wilkinson at the controls of Pilot ACE

Since the use of the punched-card equipment required the use of an operator, it encouraged user participation generally, and this was a distinctive feature of Pilot ACE operation. For example, various methods of accelerating the convergence of matrix iterative processes were left under the control of operators, and the skill with which these stratagems were used by young women with no more than high school mathematics qualifications was most impressive. Speaking for myself I gained a great deal of experience from user participation, and it was this that led to my own conversion to backward error analysis.

9 Backward Error Analysis

A description of Wilkinson's development of backward error analysis can be found at Hammarling and Higham [10, 2019], so I won't repeat it here. I do, though, want to reiterate that Wilkinson did not claim to be the first person to perform a backward error analysis. But he was certainly the person who developed the theory of backward error analysis as described in his landmark 1963 book, Rounding Errors in Algebraic Processes, [27]. In later years Wilkinson rather regretted the somewhat formal nature of his book, but at the time he felt the need to convince pure mathematicians that it was a proper rigorous theory. He was tempted to re-write the book, but never found the time to do so. Two articles which more reflect the style he would have liked to use are [35, 1986] and [34, 1985].

Two years after Rounding Errors in Algebraic Processes, Wilkinson's seminal book on the Algebraic Eigenvalue Problem appeared, [28, 1965]. As well as describing numerically stable algorithms for the solution of eigenvalue problems, it significantly extended Wilkinson's backward error results. It was identified as one of the 100 most-cited mathematics books in the period 1976–1980 and one of the most-cited mathematics publications in the period 1961–1972, Garfield [8, 1984].

10 Influence on Numerically Stable Software

The Algebraic Eigenvalue Problem had a big influence on the development of numerical software for both the solution of linear equations and the eigenvalue problem. Although he was tempted to include detailed algorithms, Wilkinson decided that it was at that stage too prohibitive so, to quote from the Preface:

Accordingly I have used the language of classical mathematics but have adopted a form of words which facilitates translation into ALGOL and related computer languages.

However Wilkinson, with Christian Reinsch¹⁷, later edited an influential book that has become known simply as "The Handbook", [37, 1971]; a collection of programs, written

¹⁷https://de.wikipedia.org/wiki/Christian_Reinsch (in German).

in Algol 60¹⁸, for solving a variety of linear algebra problems. Many of the programs had been pre-published in Numerische Mathematik. The programs were accompanied by proper documentation including background and applicability, description of parameters, discussion of numerical properties and error bounds, and test results.

Wilkinson always regretted that he had allowed Christian Reinsch to put Wilkinson's name first, the only publication of Wilkinson's where the authors are not in alphabetical order.

Most of the eigenvalue routines in The Handbook were directly translated into Fortran 66 under the EISPACK project, Smith, Boyle, Garbow, Ikebe, Klema, and Moler [15, 1974]. Algol 60 stored arrays by row, whereas Fortran stores by column, so the resulting Fortran routines could suffer serious cache misses. Many of the The Handbook routines where also used in the NAG Library, which at that time had both Algol 60 and Fortran 66 versions. At the time of writing, the NAG Library Documentation still has the Forward contributed by Wilkinson and Leslie Fox in 1975¹⁹.

Wilkinson continued to support NAG and would attend meetings of the NAG linear algebra group, usually held at NPL. He would even write notes on topics such as the singular value decomposition for the meetings²⁰. He also strongly advocated that the Library include iterative refinement in the linear equation solvers. On 14th July, 1989 the NAG office was officially opened as Wilkinson House by the then Member of the European Parliament, James Ellis, and Heather Wilkinson. See Figure 3, where Brian Ford is presenting flowers to Heather and Figure 4 showing the memorial plaque.

The LINPACK project implemented software for linear equation and least squares problems, but did not rely on The Handbook code and accessed the arrays by column wherever possible, Dongarra, Bunch, Moler, and Stewart [4, 1978]. Nevertheless, the Preface includes the statement "Finally, special thanks go to Jim Wilkinson, whose generous assistance has benefited the project at every stage."

Figure 5 shows Jim Wilkinson explaining the intricacies of cricket to Cleve Moler in the 1970s, possibly at an EISPACK project, or LINPACK project meeting. See Cleve Moler's blog at: https://blogs.mathworks.com/cleve/2013/02/18/jim-wilkinson/.

The LAPACK Users' Guide, first released in 1992, has the Dedication: "This work is dedicated to Jim Wilkinson whose ideas and spirit have given us inspiration and influenced the project at every turn." Anderson, Bai, Bischof, Blackford, Demmel, Dongarra, Du Croz, Greenbaum, Hammarling, McKenney, and Sorensen [1].

Nick Higham's wonderful book "Accuracy and Stability of Numerical Algorithms" also has a dedication to Wilkinson, alongside Alan Turing, [12, 2002].

¹⁸Two things probably led to the demise of Algol 60, it did not define input/output and, unlike Fortran, it did not have the support of IBM. It was, though, a very nice language and I used it for my first program in 1965 on an Elliott 803, using a compiler written by Tony Hoare and colleagues, Hoare [13, 1981].

¹⁹https://www.nag.co.uk/numeric/fl/nagdoc_latest/html/frontmatter/foreword.html.

 $^{^{20}}$ For example, [18; 19; 20; 21]



Figure 3: Wilkinson House Opening



Figure 4: Wilkinson House Plaque



Figure 5: Cricket at Argonne!

11 Gatlinburg and Householder Symposia

An important series of conferences was started in Gatlinburg, Tennessee in 1961, organised by Alston Householder and devoted to matrix computations and linear algebra. Wilkinson was an important part of those early conferences and, according to Josef Stoer, had a strong influence on the meetings. Stoer has written a nice article on the history and influence of the conferences in the ILAS Bulletin, IMAGE 46, Spring 2011 – see: http://www.maths. manchester.ac.uk/~higham/conferences/householder/image46_stoer.pdf²¹.

Gatlinburg VIII was held in Oxford in 1981 and organised by Leslie Fox and Jim Wilkinson. Fox and Wilkinson first met when Wilkinson joined NPL and became lifelong friends. Thanks to Wilkinson, I was invited to Gatlinburg VIII, my first Gatlinburg conference. It was quite an experience meeting so many well known numerical analysts, most of whom I only knew through their books, or papers.

Figure 6 shows the attendees of Gatlinburg VIII outside Lady Margaret Hall and Figure 7 shows Leslie Fox, Vera Kublanovskaya²², and Heather and Jim Wilkinson.



Figure 6: Attendees at Gatlinburg VIII

The first four conferences were all held in Gatlinburg and organised by Alston Householder, with Frank Olver co-organising the second conference. In honour of Householder,

²¹The complete IMAGE 46 can be found on the ILAS web site at: https://www.ilasic.org/.

²²Vera Kublanovskaya independently developed the QR algorithm.



Figure 7: Fox, Kublanovskya and the Wilkinsons at Gatlinburg VIII

the eleventh symposium, held in Tylösand, Sweden, was renamed the Householder Symposium.

12 Memorial Conference Proceedings

In July 1987 a conference, organised by NAG and NPL, on Advances in Reliable Numerical Computation was held at NPL, dedicated to Wilkinson. The opening address was given by Gene Golub on "The influence of Jim Wilkinson on his younger colleagues" and Leslie Fox²³ gave the after-dinner speech on "Jim Wilkinson: some after-dinner sentiments". All the speakers paid their own personal tributes to Wilkinson during their presentations and the proceedings were published as Cox and Hammarling [3, 1990]. The conference was a very fitting memorial to Wilkinson and the contributions are still very well worth reading.

The Wilkinson Prize for numerical software, mentioned in Section 4, was announced by Brian Ford at the conference.

To close this article I quote the final paragraphs of Gene Golub's and Leslie Fox's tribute:

Jim Wilkinson was an extraordinary scientist who created and developed a field. I think of myself as someone who applies the principles that were handed down by him in a variety of applications. The numerical linear algebra community was fortunate to have the inspiration of this great and good man. Not only did he provide scientific leadership but he was a wonderful man of unusual warmth, wit and kindness. I miss him greatly. Gene Golub.

So this tells us what our toast should be. In our gratitude for all Jim's remarkable achievements in both academic and social affairs, and remembering both how much he wanted to do and support he gave to those who were starting to do it, I suggest, and I think Jim would have liked this, that we rise to our feet and drink to 'The Numerical Future'. Leslie Fox.

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²³Leslie Fox also wrote the Royal Society biographical memoir on Wilkinson, [5, 1987]. Beresford Parlett wrote a nice article about the contribution of Wilkinson to numerical analysis, [14, 1990].

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