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THE DEVELOPMENT OF A NOVEL METHODOLOGY FOR TOMOGRAPHIC PHOTOELASTICITY

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ABSTRACT

A description is given of the current development of a novel, non-destructive, experimental method which allows the determination of 3-D stress distributions in birefringent materials. A conventional polariscope is used through which to view the model with the addition of a component positioning device. A minimal set of measurements has been devised which can be easily and efficiently realized in an actual laboratory arrangement, and which produces precisely the sufficient amount of data in order to tomographically reconstruct the full stress tensor within the bulk matter of the medium.

The paper includes an outline of the mathematical procedure, the results of numerical tests of the algorithms and a discussion of further work necessary to realise a truly practical technique.

1. INTRODUCTION

With the emergence of numerical modelling techniques over the last 40 years, the use of experimental analysis in engineering product design and development has declined. The main reasons for this decline are the cost and time involved in performing experiments compared to numerical analysis. However there is an increasing awareness that numerical results require validation and therefore experimental methods need to be developed which can measure the strains within three-dimensional components in a similar timescale to results being generated by numerical models.

In order to evaluate the internal stresses in models of 3-D components, stress freezing [1] is currently the only experimental technique routinely used. The major disadvantage to this photoelastic method is the requirement for the model to be sectioned for two dimensional analysis, consequently destroying the model. Even using the latest automated photoelastic techniques, the time needed to analyse the sections of the model is prohibitively long. If this need for sectioning could be eliminated then a single model could be used for several loading conditions and the analysis time reduced significantly. One solution to this problem may be the combination of photoelasticity with tomography.

In traditional hard field tomography, a certain radiation is passed through a section of the body and a property of this radiation (e.g. intensity, phase etc), is measured. The internal structure of the object can be reconstructed from data measured at many different views from around the body, in terms of the Radon transform. This mathematics is well understood for scalar fields, however the strain field which produces the birefringent effect is a tensor quantity rather than a scalar quantity as usually encountered in tomography. Moreover the integral equation involved is non- linear and couples the components of the tensor. Although there have been proposed systems for 3-D photoelastic tomography none has adequately addressed the issue of collecting sufficient data for reconstruction of the full stress tensor from optical measurements. Andrienko and Dubovikov[2] describe a scheme but this is not workable in practise. Doyle and Danyluk[3] and Aben *et al*[4] have until recently concentrated on axisymmetric problems. Sharafutdinov[5] gives a highly theoretical and accurate description of a set of sufficient data for the linear, quasi-isotropic photoelastic tomography problem, but it is still not stated how this would be realised in practise. Aben *et al* [6] use the Radon transform inversion on experimental data, and reconstruct one normal stress component for one plane. However it is stated that by rotating the specimen about different axes the difference of the normal stress components in any plane could be determined [7] .

A novel, non-destructive, experimental method, which allows the determination of 3-D stress distributions in birefringent materials is under development and this paper describes the progress made to date. A minimal set of measurements has been devised which can be easily and efficiently realized in an actual laboratory arrangement, and which produces precisely the sufficient amount of data in order to tomographically reconstruct the full stress tensor within the bulk matter of the medium. This paper outlines the mathematical development and the proposed experimental procedure for the methodology. A numerical simulation has been carried out to test the methodology and the results are also presented.

2. MATHEMATICAL DEVELOPMENT OF THE METHODOLOGY

In this paper we present a novel method to reconstruct the stress tensor from polarization transformation data obtained by tomographical methods. Unlike previous efforts in Integrated Photoelasticity we do not attempt to determine the stress tensor directly; rather, we first reconstruct the **dielectric** tensor inside the specimen from polarization transformation data, and only in a second step compute the stress tensor from the dielectric tensor by means of a stress-optical law [8], which links stress, strain and permittivity in mutual linear relations. The new method is based on the seminal work of Sharafutdinov [5] who formulated an abstract, and highly mathematical, framework of “Integral Geometry”, dealing with the reconstruction of tensor quantities on higher-dimensional spaces. Our own method is an adaptation and partial reformulation of this theory. The mathematical details are too lengthy for these proceedings, and these details, as well as a derivation of the equations satisfied by the anisotropy tensor in the limit of Geometrical Optics and weak anisotropy, may be found elsewhere [9]. In this paper it was proven that the linearized inverse problem of the determination of the dielectric tensor, in the “weak-anisotropy” limit, could be reduced to six measurement cycles, each cycle employing a two-dimensional Radon inversion, which yielded altogether six diagonal elements $A(\eta,\eta)$ of the anisotropy tensor A. The crucial point in the method is the careful selection of the six unit vectors η which specify the orientation of the planes along which the light rays intersect the specimen. Any vector η determines a collection of planes perpendicular to it; within each plane we scan the object by polarized light rays passing through this plane and measure the characteristic parameters [10], for all angles in this plane. From these parameters we can compute the unitary transfer matrices, which describe the polarization transformation of the light, up to a global phase. In the linear approximation at least, this global phase can also be determined [9]. A recent exposition of the relation between the equivalent optical model, describing the optical light path, the Stokes parameters of the light, and the characteristic parameters to be measured was given in Ref. [11]. In the linearized inverse problem, the (η,η) component of the transfer matrix can then be expressed as the two-dimensional Radon transform of the tensor component $A(\eta,\eta)$; but, since the latter is a rotational scalar with respect to the given plane it can immediately be computed by Radon inversion using a filtered-back projection algorithm [12]. If this procedure is repeated for six carefully chosen directions η we can determine the anisotropy tensor A, and thus the full dielectric tensor, within the whole interior of the photoelastic object; in this last step we utilize the so-called polarization identity, well known from basic Linear Algebra.

Once the dielectric tensor is reconstructed, the stress tensor can then be obtained from a knowledge of the stress-optical law for the given photoelastic material.

The crucial advantage of the new method is that the equations of the linearized inverse problem now contain the tensor components of the anisotropy tensor A directly; whereas, in previous attempts [7,13], the inverse problem had always been formulated in terms of the stress tensor. However, the latter enters the linearized problem only through the *differences* between the principal stress components, so that the stress cannot be reconstructed directly, except for stress configurations exhibiting a certain degree of symmetry (typically axial symmetry). In contrast, our method is capable of reconstructing arbitrary dielectric tensors, hence arbitrary stress tensors, as long as the degree of anisotropy is reasonably small. For a photoelastic specimen this means that the dielectric tensor should deviate weakly from the homogeneous isotropic permittivity of the unloaded material. The precise specification of this condition is found in [9].

3. EXPERIMENTAL DESIGN

It is proposed that a conventional polariscope is used to collect the data needed for the tomographic reconstruction with the addition of a component positioning device which is shown in Figure 1.

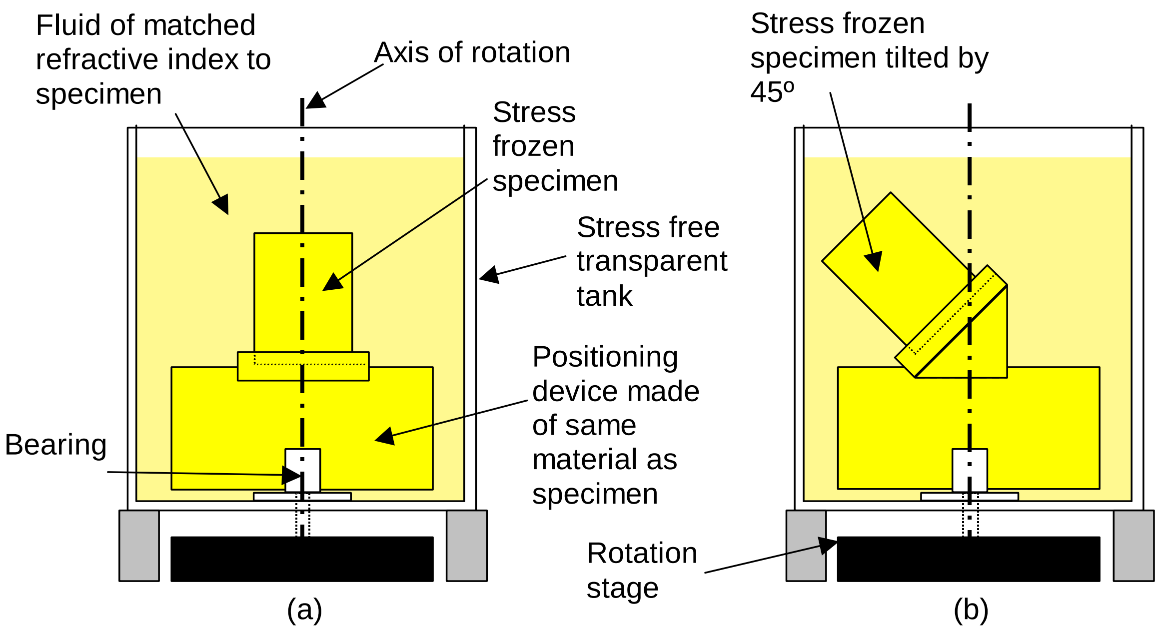


Figure 1 Proposed design for the component positioning device, showing the specimen tilted by a) 0° and b) 45°. The immersion tank is placed in the field of view of a conventional polariscope

As described above, in order to collect sufficient data, the specimen is required to be rotated within the field of view for six different axes of rotation. It is proposed that this can be practically achieved by having one axis of rotation and re-positioning the specimen with respect to this axis. It has been found that it is necessary to tilt the object with respect to the axis of rotation and for each tilt angle rotate the specimen about each of its three orthogonal axes. The optimum tilt angles which result in sufficient data are 0º and 45º to the axis of rotation, resulting in six orientations of the specimen (Figure 2) and consequently six sets of data in total. Figures 1 (a) and (b) show the specimen placed on a component positioning device orientated at a tilt angle of 0° and 45° respectively. The rotation stage located below the tank will drive the rotation of the component positioning device. It is essential that, when it is tilted, the component positioning device must not block the light path and therefore should be manufactured from a transparent material of known refractive index. If it is made from the same material as the birefringent specimen, this will further simplify the mathematics. The immersion tank is filled with a fluid of matched refractive index to the specimen to prevent refraction of the light at the specimen surface.

3.1 Data collection

The specimen is rotated 180° about the axis of rotation and the three characteristic photoelastic parameters [10] are determined at discrete intervals during this rotation. A Fourier polarimetry method has been chosen to do this [14], where the analyser and polariser are rotated simultaneously at a fixed ratio of 3:1 over a 360° revolution of the polariser and a number of intensity images are recorded at discrete intervals. The detected intensity signal can be expanded into a Fourier series using a fast Fourier transform of the images so that the Fourier coefficients can be found. These Fourier coefficients can be described in terms of the characteristic parameters. The polariser and analyser of the polariscope are controlled by the same control unit as the rotation stage, and therefore all motion is synchronised. Once the characteristic parameters are determined for each orientation of the specimen these are then processed using the procedure described above.

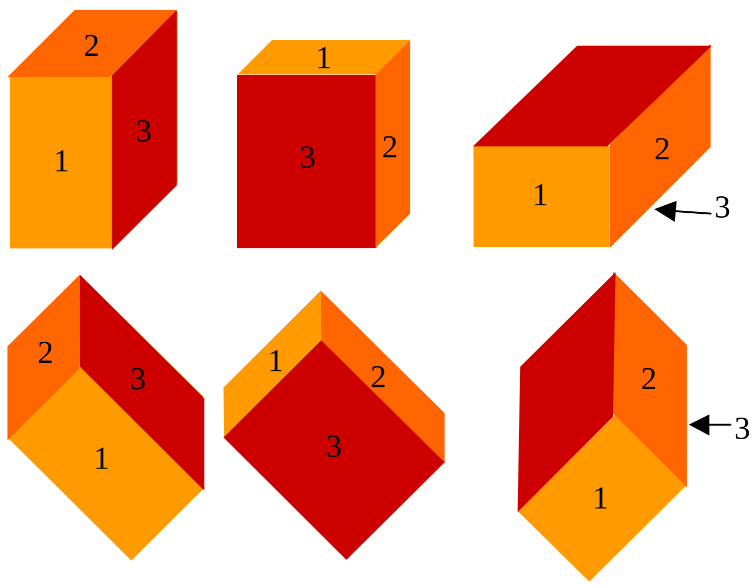


Figure 2 The six orientations of the specimen relative to a vertical axis of rotation.

4. NUMERICAL SIMULATION

Since the experimental procedure is under development, the methodology was tested numerically by simulating the strain field for an axially loaded cylinder, shown in Figure 3, calculating the strain tensor analytically assuming linear elasticity and an elastically isotropic material. The forward problem was solved using a standard ODE solver along each ray. The linear approximation was *not* assumed for the forward calculation. The reconstruction was performed using a standard ramp filtered back projection [12]. For this test no attempt was made to simulate noise in the data, although the inverse Radon transform is known to be mildly ill posed so we would expect reconstructions to degrade significantly with measurement error unless some smoothness of the solution was assumed *a priori*.

The results of the reconstruction algorithm shown in Figure 4 are typical of the results for many planes which were analysed. Other results have also been presented elsewhere [9,15] For purposes of illustration, two examples of reconstructions, are shown. In each case the component of the permittivity tensor normal to the plane intersecting the cylinder is presented. Figure 4 a) shows the original tensor for the plane shown in Figure 3. Figures 4 b) and c) show the reconstruction of the same plane. In Figure 4 b) the resolution is 50x50 pixels based on 36 scans, i.e. 5º intervals between each increment of rotation, while in Figure 4 b) the resolution is 254x254 pixels, based on 180 scans of the object, i.e. 1º intervals. The latter is almost indistinguishable at this resolution to the original simulated permittivity tensor component in Figure 4 a).

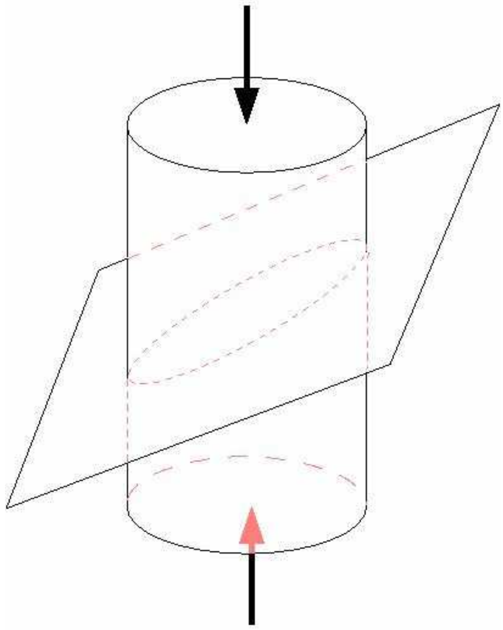
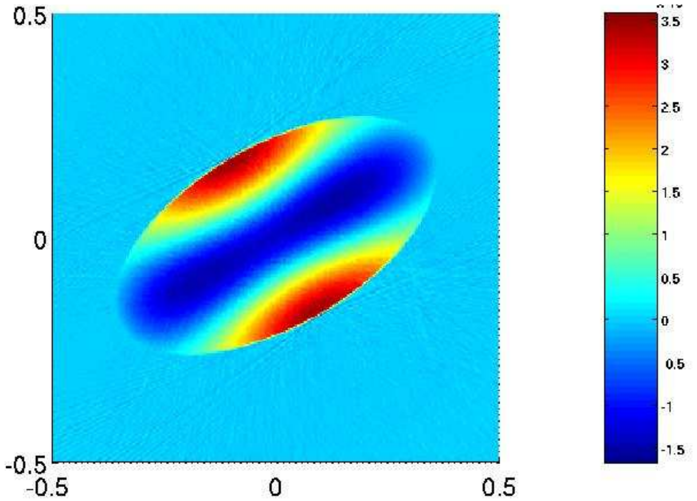
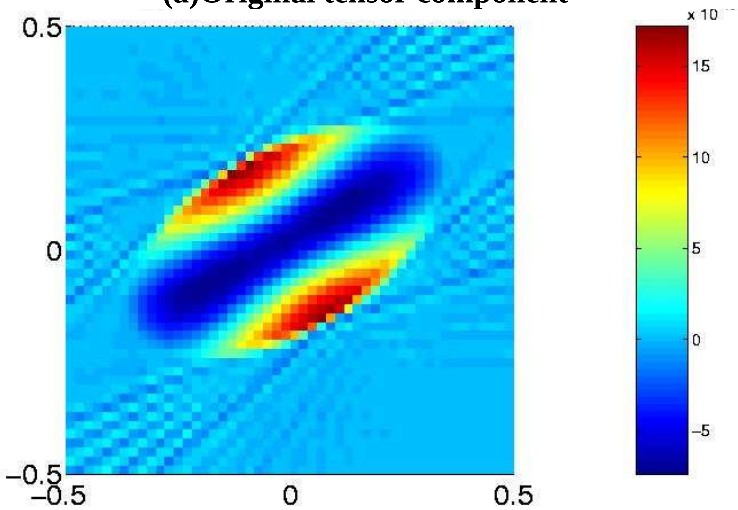


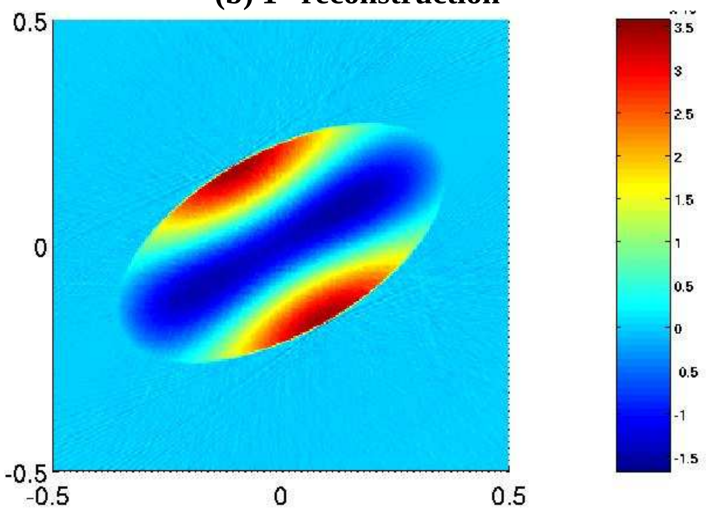
Figure 3 A cylinder loaded in axial compression which was used for the numerical simulation. The permittivity tensor for the plane indicated is shown in Figure 4



(a)Original tensor component



(b) 1st reconstruction



(c) 2nd reconstruction

Figure 4. Results of reconstruction for simulated data. (a) The original tensor component (b) A reconstruction using 5º intervals of rotation on a 50×50 grid, showing artefacts typical of an inverse Radon transform with incomplete data. (c) Reconstruction on a 256×256 pixel grid using scans at 1° intervals of rotation.

5. DISCUSSION

With further experimental development, it is considered that the proposed methodology will be a useful, practical tool in the verification of numerical models. Although this method does not eliminate the need for stress freezing, it does eliminate the need for the sectioning of three-dimensional photoelastic models, thereby allowing a single model to be used to examine several loading conditions. However, it must be recognised that this is an ongoing project and that paper presents the work carried out so far, and several obstacles must be overcome before the realisation of a truly practical experimental method.

In standard photoelasticity the relative stress optic coefficient is measured and therefore only the difference in principal stresses may be obtained directly [16]. The proposed method would require the determination of the absolute stress optic coefficients, c_1 and c_2 in order to determine the principal stresses from the dielectric tensor. Measurements of absolute retardations for such calibration purposes have been made using a Mach-Zehnder interferometer [17], but are difficult and time consuming. We admit that the requirement to make such measurements is a drawback to the proposed technique and alternative measurement methods are currently being explored.

The success of reconstructing simulated data will aid the selection of resolution and rotation intervals required for the experimental implementation. Figure 4b) shows that a relatively coarse resolution and interval of rotation can still give recognisable stress tensors therefore data could possibly be collected much faster than originally thought and at a lower resolution. This remains to be tested.

If tomographic photoelasticity is to be implemented in industrial applications, then the limitation of the proposed methodology to weak anisotropy must be overcome. For higher strain fields, where the linear approximation is not valid, the forward solver used for this simulation, together with the inverse solver for the linearized problem (assuming an isotropic background before perturbation) could be used in a Newton-Kantarovich [18] method for non-linear reconstruction.

6. CONCLUSIONS

A numerical algorithm to reconstruct the full 3-D strain field for the general non-symmetric case, under the assumption of sufficiently small strain has been implemented. A possible experimental system capable of collecting the data required by this algorithm has been described and further issues to be considered have been discussed.

7. ACKNOWLEDGEMENTS

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