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When do the iterative reconstruction methods become worth the effort?

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***Abstract*—A driving force for the development of new reconstruction algorithms is to achieve better quality images using less information (lower dose, fewer projections, in less time), but under what circumstances do iterative methods become worth the effort? In this paper we propose a framework that enables the performance of reconstruction algorithms to be mapped. Such a framework allows fair comparisons to be made, providing insights into experimental acquisition strategies and methods of quantifying the quality of reconstructions, and identifying the sweet spot for different algorithms.**

In the CT imaging community, the challenge is to be able to produce the best quality images with the least amount of information. Depending on the application, this *information* could be a series of quickly acquired projections if we wish to capture rapid changes in the sample, or low dose exposures given to a patient during the scans; or it could be a limited angles of illumination due to the constraints of the hardware, the sample, or restrictions to minimize the computational memory requirements.

The science of reconstructing a 3D volume from 2D projections is a problem with many possible solutions. Even in the case of sufficient data, the solutions can be unstable. Additionally, these solutions are sensitive to small changes in the measured data (noise due to modeling or experimental errors), which means that even a noisy image can qualify as a feasible solution. This solution can be an image of the location of a landmine in the ground, or detection of a weapon in a bag, or used to diagnose cancerous cells. This becomes an issue when the reconstruction techniques do not converge to a better solution than a noisy image. Therefore it is natural that we consider the data to be measured, any prior information about the problem we can make use of, and the methods for reconstructing high quality images. However before thinking about the reconstruction algorithms, we must first consider whether the information we have is useful. This can be done by characterizing the *cost of measure*. This is simply a variable specific to the application. In our case, this could be the available acquisition time, or perhaps the level of dose we use to scan a patient. It is clear that, whatever the application is, we do not want a dataset comprising just one projection acquired over a long period of time (strong signal), or many projections in very short periods (weak signal). We want to be able to determine what we need to know, and plan experiments accordingly. Taking into account the important variables in CT, we propose a map to guide the reader when planning experiments. This map displays aspects of the information content of the dataset with the abscissa quantifying the number of projections and the ordinate the number of photons collected (proportional to the number of frames acquired per projection times the number of projections). In this space a given experimental strategy is a point on the map. If the quality of the reconstructed image can be expressed as a metric, then it can be used to map the capability of a given algorithm highlight acquisition regimes over which it maybe deemed acceptable. Similarly regimes over which different algorithms are beneficial can be identified. Conversely, acquisition strategies can be identified to achieve a given performance in the least time or dose.

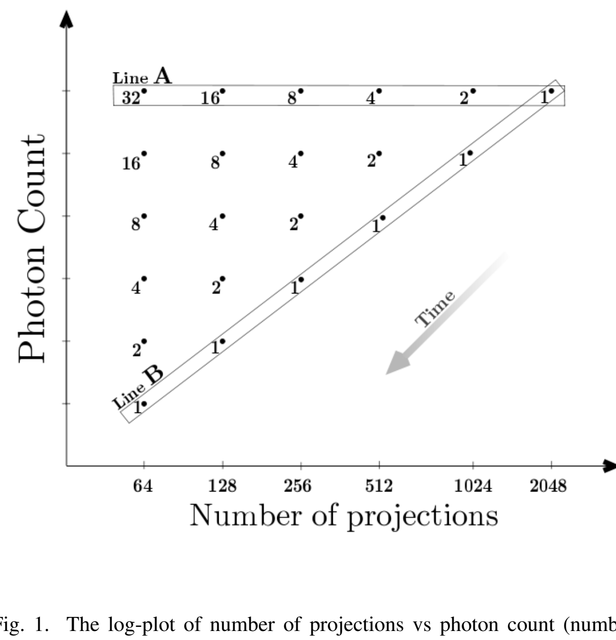

Fig. 1. The log-plot of number of projections vs photon count (number of projections $\times$ number of frames). The numbers on the chart refer to the number of frames for scans lying on LINES A and B.

From this map it is clear that if we want to identify the performance of algorithms as a function of the number of projections, it is fairer to keep the number of photons collected constant while the number of projections is varied (LINE A in Fig. 1). By contrast it is often the case that in practice the comparison is made using one frame for each projection (LINE B) which convolves the decreasing number of projections with the decreasing signal, influencing our conclusions. The

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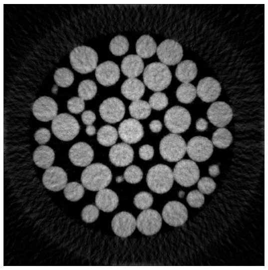
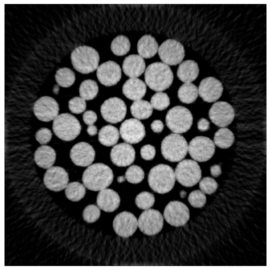
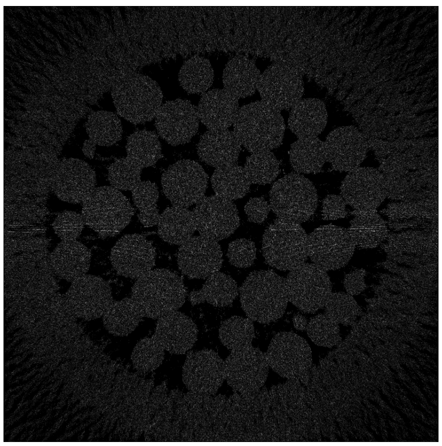
difference in reconstructed images for both LINES A and B is illustrated in Fig. 2(c), where we have used the SophiaBeads 128 dataset with 1 frame (lying on LINE B) and 16 frames (on LINE A). We introduce the SophiaBeads Datasets in the next section.



(a) 128 projections, 1 frame.



(b) 128 projections, 16 frames.



(c) Difference between the reconstructed results.

Fig. 2. Highlighting the difference in reconstructions with the traditional approach following LINE B (top left), and keeping the cost of measure constant by following LINE A (top right). The eliminated noise is shown in the image (bottom). Reconstructions are obtained using CGLS.

Our ultimate aim is to introduce a framework for designing experiments and choosing appropriate reconstruction methods via fair comparisons. In this paper, we wish to discuss this aspect of our work, and support our logic with reconstructed results. Before concluding the paper, we explore ways of quantifying results using a real dataset.

I. EXPERIMENTAL DESIGN AND QUANTIFICATION METHODS

We have established an experimental glass bead pack dataset, [5], based on the above framework acquiring 1 frame for each of 2048 projections; 2 frames at 1024, 4 frames at 512, 8 frames at 256, 16 frames at 128 and 32 frames for 64 projections (see points lying on LINE A in Fig. 1). This enables a wide range of algorithm comparisons and information content optimizations to be examined. In this paper we examine the performance of algorithms along LINE A, namely we compare the performance of algorithms using different numbers of frames but at a constant signal.

The experiment dataset is called SophiaBeads, available as part of the SophiaBeads Datasets Project. More information on the sample, data acquisition and quantitative analysis of the reconstructions can be found in [6, 4]. We have chosen a beads problem because it is easy to make the dimensions of the solid spheres precisely known and the problem is representative of many X-ray imaging problems [7]. A key element of the beads problems is that the samples often consist of only beads and air, making them suitable for studying 'porous channels' (bottlenecks) and 'touching of beads'. These are important characterizations for studying segmentation techniques or discrete CT algorithms.

Our motivation for using SophiaBeads Datasets in particular is that we know what the reconstructions should look like. We know that the beads are of one size[1], and thus the following items can be considered when quantifying our results:

I. **Volume of the beads**[2]: The range of expected volume of each bead is known.
II. **Shape of the beads**[2]: We can parameterize how close (in shape) a reconstructed bead is to a perfect sphere.
III. **Circular cross-section of the beads**[3]: Because the beads are (nearly) perfectly spherical, each bead should have the same[1] diameters or radii in all axes (i.e. a cross-section of each bead should be a perfect circle).
VI. **Smoothness of the beads**[3]: boundary of each bead should be smooth.

II. COMPARISON OF RECONSTRUCTION METHODS

In this section, we present the 2D reconstructions of the SophiaBeads Datasets, using FDK [8], CGLS [3] and SART [2]. To examine the reconstructions in detail, we focus on a central window of the reconstructed slices (see Fig. 3).

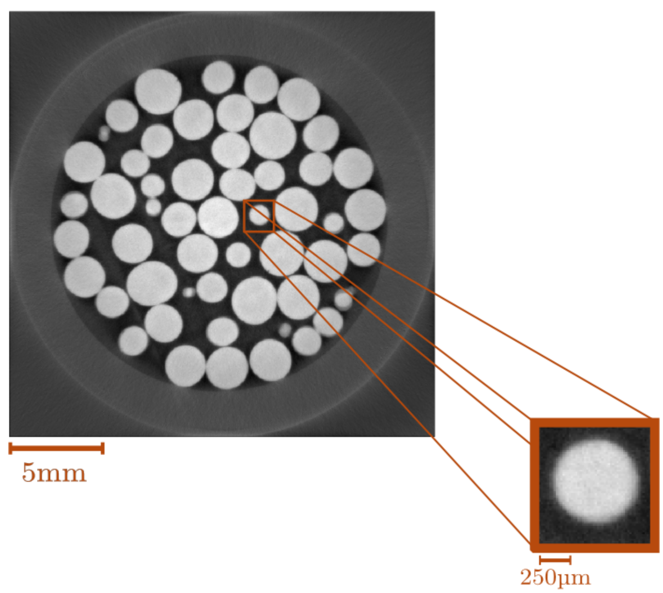


Fig. 3. 2D CGLS reconstruction of 2048 projections, and its center window.

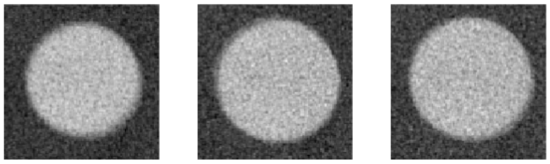
[1]The size of the beads is normally distributed with a mean of 2.5mm (in diameter), and a standard deviation of 0.01mm (or 100μm). This means that even though most beads in 2D will look like perfect circles, there will be a proportion of them that are egg-shaped.

[3]Suitable for 2D and 3D reconstructions.

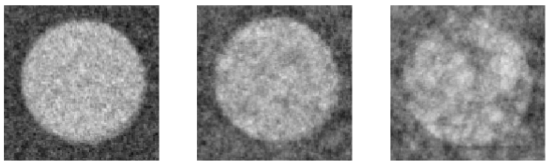
[2]Requires 3D reconstructions.

FDK Results

FDK [8] is the standard approach employed by most commercial scanners. Results below are obtained using the in-house implementation of FDK.



(a) 2048. (b) 1024. (c) 512.



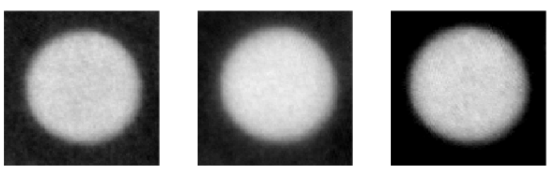
(d) 256. (e) 128. (f) 64.

Fig. 4. 2D FDK reconstructions of each SophiaBeads dataset.

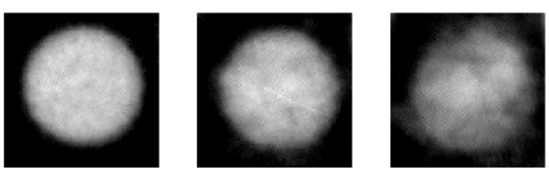
One can observe line artefacts and loss of contrast in the reconstructed images in scans with fewer projections. In particular in Fig. 4(f), the bead is almost unidentifiable due to loss of definition in shape.

CGLS Results

This is the Conjugate Gradient method modified for non-square systems such as the CT problem, as explained in [3].



(a) 2048. (b) 1024. (c) 512.



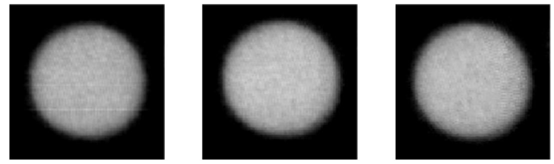
(d) 256. (e) 128. (f) 64.

Fig. 5. CGLS reconstructions at iteration 12.

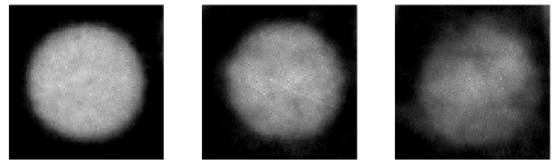
The method is implemented in MATLAB R2014b, with the forward and back projector codes written in C. This method is also used in the SophiaBeads tutorials [4]. The number of iterations is fixed at 12. We observe increase in blur and loss of definition of bead shape in scans with fewer projections.

SART Results

As the third example, we present results using a popular method from the family of algebraic reconstruction techniques [2]. For these runs, we performed 200 sweeps with a relaxation factor chosen as 0.8. Just as with CGLS, we have implemented and plotted results in MATLAB with forward and back projectors implemented in C.



(a) 2048. (b) 1024. (c) 512.



(d) 256. (e) 128. (f) 64.

Fig. 6. SART reconstructions with 200 sweeps, and relaxation factor $\omega = 0.8$.

We observe relatively greater loss of definition in the shape and contrast (compared to CGLS), as the number of projections decrease.

Quantifying the SophiaBeads Reconstructions

To evaluate our results we use the quantification item II (henceforth referred to as SHAPE3D). For the analysis, we have used built-in image-measuring techniques in Avizo Fire 8, where the reconstructed volume is read by Avizo (see the quantification tutorial in [4]).

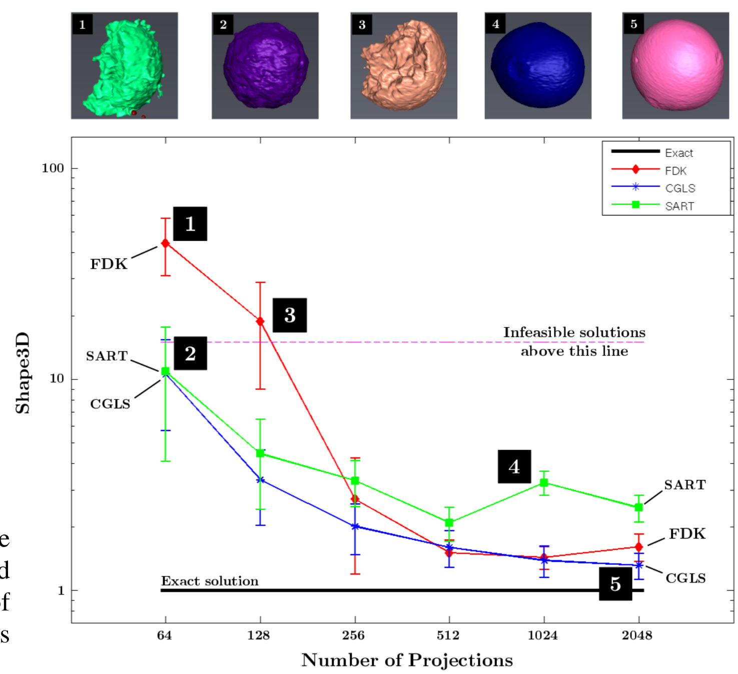


Fig. 7. Results of the SHAPE3D analysis plotted with errorbars in MATLAB.

Fig. 7 is the log-plot of the mean and the standard deviation of each reconstructed volume. The image-measuring technique in Avizo attempts to fit the each bead to a unit sphere, and parameterizes how close that bead is to a perfect fit (more details in the Avizo user manual [1]). If the reconstructed beads fit the model sphere perfectly, then Avizo outputs 1, so this is taken as the exact answer. Anything above the dashed line can be dismissed as an infeasible solution. From this it is clear that FDK at 64 iterations is a poor choice as a reconstruction method. For datasets with 256 projections or more, we see all three methods giving similar results with small standard deviations.

III. Conclusions

As the CT community, we welcome novel ideas for iterative reconstruction methods for better quality reconstructions. We spend time on defining our problems, creating and testing ideas, and developing algorithms. Yet we still struggle to answer this simple question: When should we be using iterative methods?

In this paper, we offered a strategy to help us answer this question by introducing a map to plan trials and across which the performance of various algorithms can be charted. Here we examined the effect of altering the number of projections whilst keeping the photon count constant. This has shown that iterative methods deal better with datasets with fewer projections, whereas the FDK method is adequate for scans with 256 projections or higher.

The SophiaBeads Datasets [5] were acquired in such a way that it allows a multi-faceted exploration of the effect of decreasing the information content on the performance of reconstruction algorithms as outlined in Fig. 1. Another key advantage of the SophiaBeads Datasets was that many aspects of the actual 3D object are precisely known, enabling us to quantify algorithm performance. It is noteworthy that the framework in Fig. 1 and the SophiaBeads Datasets allow a wide range of experimental strategies to be simulated (minimum time, dose, number of projections) and the limits of the algorithms delineated or the most appropriate one identified. We would like to note here that there was no prior information used in these reconstructions, which is outside the scope of this discussion. However, improvements in images and changes in fewer projection artefacts when prior information is used are interesting topics that deserve further discussion. In addition, because the beads problem is amenable to discrete tomography, an algorithm (with a suitable prior information) may outperform the current algorithms in quality and speed.

Acknowledgment

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