

Stochastic Reachability Analysis of Hybrid Systems

Bujorianu, L. M.

2012

MIMS EPrint: **2012.43**

Manchester Institute for Mathematical Sciences
School of Mathematics

The University of Manchester

Reports available from: <http://eprints.maths.manchester.ac.uk/>

And by contacting:
The MIMS Secretary
School of Mathematics
The University of Manchester
Manchester, M13 9PL, UK

ISSN 1749-9097

Contents

1	Introduction	1
2	Markov Models	5
2.1	Overview	5
2.2	Discrete Space Markov Models	5
2.2.1	Discrete Time Markov Chains	5
2.2.2	Continuous Time Markov Chains	7
2.3	Continuous Space Markov Models	11
2.3.1	Strong Markov Processes	14
2.3.2	Continuous Processes	15
2.3.3	Discontinuous Processes	18
2.4	Markov Process Characterisations	21
2.4.1	Operator Methods	21
2.4.2	Kolmogorov Equations	25
2.4.3	Martingale Problem and Extended Generator	26
2.5	Some Remarks	28
3	Hybrid Systems: Deterministic to Stochastic Perspectives	31
3.1	Overview	31
3.2	Deterministic Hybrid Systems	32
3.2.1	Modelling	33
3.2.2	Analysis	38
3.3	Randomness Issues when Modelling Hybrid Systems	40
3.3.1	Probabilistic Hybrid Automata	41
3.3.2	Piecewise Deterministic Markov Processes	42
3.3.3	Stochastic Hybrid Systems	45
3.3.4	Switching Diffusion Processes	47
3.3.5	General Switching Diffusion Processes	49
3.3.6	Analysis of Stochastic Hybrid Systems	51
3.4	Some Remarks	53

4 Stochastic Hybrid Systems	55
4.1 Overview	55
4.2 General Stochastic Hybrid Systems	56
4.2.1 Informal Presentation	57
4.2.2 The Mathematical Model	58
4.2.3 Formal Definitions	61
4.3 Markov String	63
4.3.1 Informal Description	63
4.3.2 The Components	64
4.3.3 Piecing out Markov Processes	69
4.3.4 Basic Properties	73
4.4 Stochastic Hybrid Processes	77
4.5 Examples of Stochastic Hybrid Systems	81
4.5.1 Single-Server Queues	81
4.5.2 A Hybrid Manufacturing System Model	82
4.5.3 A Simplified Model of a Truck with Flexible Transmission	83
4.5.4 The Stochastic Thermostat	84
4.6 Some Remarks	85
5 Stochastic Reachability Concepts	87
5.1 Overview	87
5.2 Stochastic Hybrid Control	87
5.3 Stochastic Reachability Problem	89
5.3.1 Motivation	89
5.3.2 Mathematical Formulation	91
5.4 Measurability Results	92
5.5 Different Measures Associated with Stochastic Reachability	93
5.5.1 Hitting Distributions	94
5.5.2 Exit Distributions	96
5.5.3 Occupation Measure	97
5.5.4 Occupation Time Distribution	98
5.5.5 Imprecise Probability	98
5.5.6 Réduite and Balayage	101
5.6 Some Remarks	103
6 Probabilistic Methods for Stochastic Reachability	105
6.1 Overview	105
6.2 Reachability Estimation via Hitting Times	105
6.2.1 Hitting Distributions for Markov Chains	105
6.2.2 Hitting Distributions of GSHSs	110
6.3 Reachability Estimation via Transition Semigroup	111
6.3.1 The Case of a Markov Chain	112
6.3.2 The Case of a Markov Process	113
6.3.3 The Case of a Stochastic Hybrid Process	114
6.4 Reachability Estimation via Martin Capacities	115
6.4.1 Martin Capacity	116

6.4.2	The Case of a Markov Chain	116
6.4.3	The Case of Brownian Motion	117
6.4.4	The Case of a Markov Process	117
6.5	Reachability Estimation via Martingale Theory	122
6.5.1	Method Based on the Martingale Problem	122
6.5.2	Method Based on Martingale Inequalities	124
6.5.3	Method Based on the Barrier Certificates	126
6.6	Reachability Estimation via Quadratic Forms	129
6.6.1	Target Sets	129
6.6.2	Dirichlet Forms	130
6.6.3	Induced Dirichlet Forms	130
6.6.4	Upper Bounds for Reach Set Probabilities	132
6.7	Some Remarks	133
7	Analytic Methods for Stochastic Reachability	135
7.1	Overview	135
7.2	Réduite and Optimal Stopping	135
7.3	Stochastic Reachability as an Optimal Stopping Problem	136
7.4	Optimal Stopping Problem for Borel Right Processes	137
7.5	Methods Based on Excessive Function Representations	138
7.5.1	How the Method Works for Stochastic Hybrid Systems	141
7.6	Variational Inequalities for Stochastic Reachability (I)	145
7.7	Optimal Stopping Problem as an Obstacle Problem	147
7.8	Energy Form	149
7.8.1	Energy Form Associated to a Borel Right Process	150
7.9	Variational Inequalities on Hilbert Spaces	151
7.10	Variational Inequalities for Stochastic Reachability (II)	154
7.10.1	Reachability for Diffusion Processes	155
7.10.2	Reachability for Jump Processes	156
7.11	Hamilton–Jacobi–Bellman Equations	157
7.12	Some Remarks	162
8	Statistical Methods to Stochastic Reachability	163
8.1	Overview	163
8.2	Imprecise Probabilities	164
8.2.1	Choquet Integral	165
8.2.2	Bayes’ Theorem for Capacities	165
8.3	Capacities and Stochastic Reachability	167
8.3.1	Capacities Associated to GS HS Realisations	167
8.3.2	The Generalised Bayes Rule for Stochastic Reachability	169
8.3.3	Computing Reach Set Probabilities	170
8.4	Some Remarks	172
9	Stochastic Reachability Based on Probabilistic Bisimulation	173
9.1	Overview	173
9.2	Stochastic Equivalence	174
9.3	Stochastic Bisimulation	176

9.4	Bisimulation via a Stochastic Kernel	178
9.5	Bisimulation Between Two Stochastic Processes	179
9.6	Bisimulations Preserving Reach Probabilities	182
9.7	Some Remarks	185
10	Stochastic Reachability with Constraints	187
10.1	Overview	187
10.2	Mathematical Definitions	188
10.3	Mathematical Characterisations	189
10.4	State-Constrained Reachability in a Development Setting	194
10.4.1	Multilayer Stochastic Hybrid Systems	194
10.4.2	Electrostatic Analogy	198
10.5	Further Generalisations: Randomised Targets	198
10.6	Some Remarks	202
11	Applications of Stochastic Reachability	203
11.1	Overview	203
11.2	Applications of Stochastic Reachability in Air Traffic Management	203
11.2.1	Work of Blom and Co-workers	205
11.2.2	Work of Lygeros and Co-workers	205
11.2.3	Work of Prandini and Hu	206
11.3	Applications of Stochastic Reachability in Biology	206
11.4	Some Remarks	207
Appendix	209
A.1	Ordinary Differential Equations	209
A.2	Dynamical Systems	210
A.3	Probabilistic Measure Theory	211
A.3.1	Random Variables	214
A.4	Random Processes: General Theory	220
A.5	Stochastic Models	223
A.5.1	Martingales	223
A.5.2	Markov Processes	224
References	233
Index	243



<http://www.springer.com/978-1-4471-2794-9>

Stochastic Reachability Analysis of Hybrid Systems

Bujorianu, L.M.

2012, XVII, 248 p. 14 illus., 8 in color., Hardcover

ISBN: 978-1-4471-2794-9