Human Dimension of Mathematics

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2007

MIMS EPrint: **2008.38**

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Human Dimension of Mathematics

A Mathematician's Look at
History, Philosophy, Politics of Mathematics

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Ankara, 5 April 2007

I. Social dimension of Mathematics
II. Adam Smith and mathematics
III. What is mathematics, really?
IV. Mathematics and the brain
V. What should mathematicians do?

I. Social dimension of Mathematics

Let us look at something practical.

A case study: RSA

- On of the most widely used cryptographic systems.

- Security **is not proven**.

- **Trust** in RSA is rooted in the fact that, allegedly, USA and USSR adopted it in the systems of control of compliance with the Threshold Nuclear Ban Treaty (1974).

A case study: RSA

- Breaking RSA is equivalent to breaking integers like 6, 15, 21, 35, . . . into product of smaller integers:

  $6 = 2 \times 3$, $15 = 3 \times 5$, $21 = 3 \times 7$, . . .

- The latter problem is assumed to be **very difficult**: can you decompose 391? 589?

- But no-one ever offered any definite proof that the problem is indeed difficult.

- Why could not it happen that some clever trick breaks RSA?

A case study: RSA

By the way,

$$391 = 17 \times 23$$
$$589 = 19 \times 31$$

RSA routinely uses 150-digit numbers.

A case study: RSA

- Our **faith** that RSA is secure is based on 2000 years of **collective negative experience** of mathematicians, who tried and **failed** decompose large numbers.

- It is hard to imagine anything firmer rooted in the social and cultural practices of humankind.

Disclaimer for philosophers—if there are any in the audience.

I am **not** a social constructivist: I insist that some of of mathematics associated with RSA is so deep, complex, and abstract, that its very existence cannot be explained within the social constructivism framework.

II. Adam Smith and mathematics

English banknote with a portrait of Adam Smith

The words on the note:

"The division of labour in pin manufacturing"

AN

INQUIRY

INTO THE

Nature and Causes

OF THE

WEALTH OF NATIONS.

By ADAM SMITH, LL. D. and F. R. S.

Formerly Profeffor of Moral Philofophy in the Univerfity of GLASGOW.

IN TWO VOLUMES.

VOL. I.

LONDON:

PRINTED FOR W. STRAHAN; AND T. CADELL, IN THE STRAND.

MDCCLXXVI.

13

Book I, Chapter I: Of The Division of Labour

On pin manufacturing:

> "One man draws out the wire; another straights it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on is a peculiar business; to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations."

Adam Smith's conclusion:

Separation of the pin production process into 18 operations increases the productivity by factor of **240**.

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The consequences are profound.

Mathematics built-in in a mobile phone or MP3 player is beyond understanding by most graduates from mathematics departments in British universities.

In the emerging division of **intellectual labour**, mathematics is a 21st century equivalent of sharpening a pin.

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Of course, the same is true about physics, chemistry, biology . . .

. . . although biology is perhaps not sharpening the pin but attaching a head, which, as Adam Smith remarks, in itself consists of two or three operations.

We have to admit that 95% of population do not need any mathematics beyond use of a calculator.

But what are the implications for mathematical education?

Traditional
pyramid of
mathematical
education

And this is what
we should expect
in the future
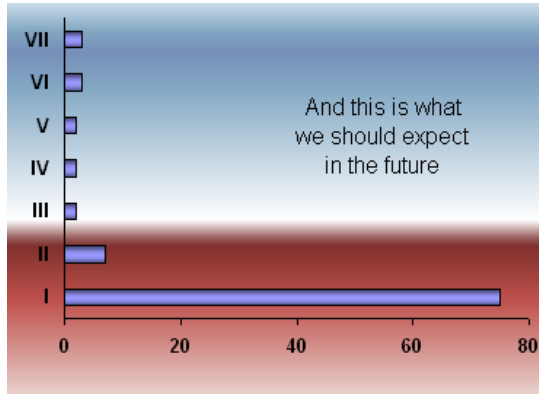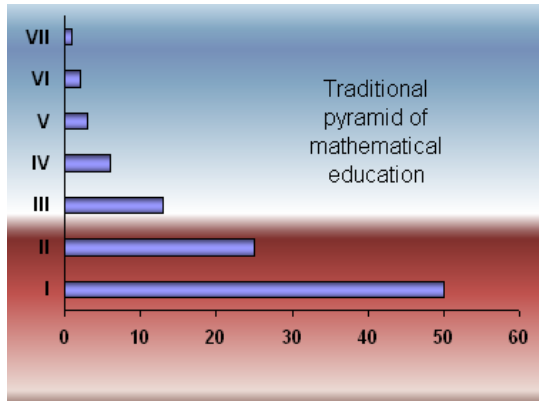
Collapse of the traditional pyramid of education

In Britain, the natural cycle of reproduction of mathematics as a cultural system and a professional community is broken.

> ...the performance of more able pupils had collapsed; the numbers taking A-level maths were falling dramatically; those with top grades were "increasingly innumerate and even ineducable"; the shortage of qualified maths teachers had reached "dangerous" levels; national test results were grossly inflated; and postgraduates with a PhD in maths from a British university were now "largely unemployable" in British universities. (The Daily Telegraph, 28 June 2005).

Rebranding mathematics

Why not rebrand mathematics as a tool of personal development and a spiritually enhancing activity?

Why not try to create an up-market brand of maths learning, for the top 5% who still need it?

Rebranding mathematics

Selective mathematical education will not work unless we know:

- What are mathematical abilities?

- What is the nature of mathematical intuition?

- What children actually do when they learn mathematics?

- What mathematicians actually do when they do mathematics?

III. What is mathematics, really?

Davis and Hersh, endorsed by Mumford:

Mathematics is

the study of mental objects with reproducible properties.

Information processing rates:

- conscious and controlled reasoning: 16 bits per second (further falling to 5 or 6 in arithmetic).

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We do mathematics by developing an elaborate system of multilayer hierarchical control of powerful sub- and unconscious modules of our mind by its tiny conscious part.

Potential selling point:

Mathematics is a method for harnessing and controlling the huge computational power of our brain.

IV. Mathematics and the brain

Why does the mirror changes left and right

...but does not change up and down?

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. . . but does not change up and down?

Answer: it does not.

Instead it changes forward/backward directions.

Changing of left and right is just a popular myth.

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Question: Why does the myth exist?

Why does the mirror changes left and right
... but does not change up and down?

Answer: it does not.

Changing of left and right is just a popular myth.

Question: Why does the myth exist?

Answer: Because we attribute to the mirror the intrinsic bilateral symmetry of our cognitive system.

Mirror writing

1 in 600 people can write by non-dominant hand in mirror script.

The rest 599 can do a weaker version of mirror writing.

Mirror writing

1 in 600 people can write by non-dominant hand in mirror script.

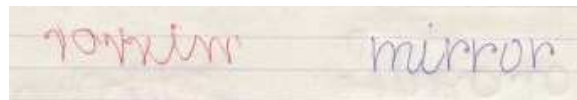The rest 599 can do a weaker version of mirror writing:

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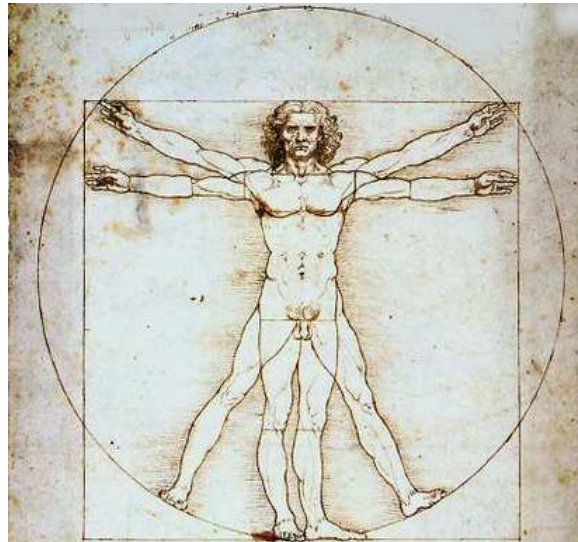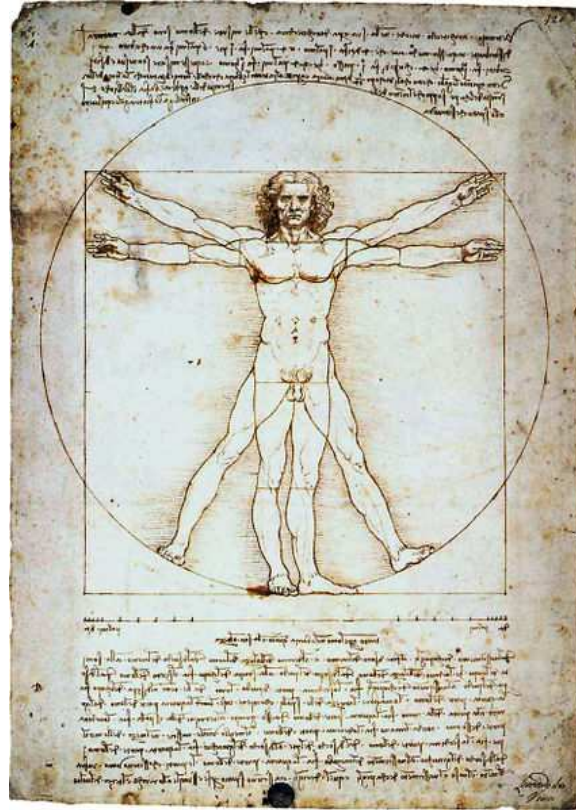The rest 599 can do a weaker version of mirror writing:

Mirror reflection:

Mirror writing: Leonardo da Vinci

Everyone know his famous study of the symmetry of human body:

It is less known that Leonardo's notes are in **mirror writing**:

Mathematical illustration: Euler's Theorem

If an orientation-preserving isometry of the affine Euclidean space $\mathbb{A}\mathbb{R}^3$ has a fixed point then it is a rotation around some axis.

Numerous psychological experiments show that Euler's Theorem is hardwired in our brain.

Coxeter's proof, verbatim:

In three dimensions, a congruent transformation that leaves a point O invariant is the product of at most three reflections: one to bring together the two x-axes, another for the y-axes, and a third (if necessary) for the z-axes.

Since the product of three reflections is opposite, a direct transformation with an invariant point O can only be the product of reflections in *two* planes through O, i.e., a rotation.

The intuition of symmetry is rooted in both visual and sensorimotor systems.

Another example of such overlap: convexity.

A symptom of interaction between sensorimotor and visual intuitions: **proof by handwaving**.

Grinding flat mirrors
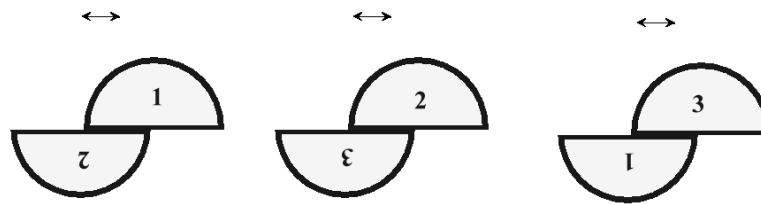
Take three pieces of glass and grind 1-st and 2-nd pieces together. Then do the same for the 2-nd and the 3-rd pieces and then for the 3-rd and 1-st pieces.

Repeat many times and all three pieces of glass will become very accurately flat. **Why?**

A rather deep mathematics is just one step away:

What surface do we get if we grind only *two* pieces of glass?

The most natural solution:

describe 3-dimensional subalgebras in the Lie algebra

$$\mathrm{Lie}\left(\mathbb{R}^3 \rtimes \mathrm{SO}(\mathbb{R})\right)$$

Of course, there are alternative solutions.

V. What should mathematicians do?

What should mathematicians do?

- Be prepared to do serious and co-ordinated political lobbying.

- Form the trade union of pin-sharpeners.

- In their daily work, pay serious attention to the human dimension of mathematics.

- Work with mass media, Internet communities, parents and children.

- Build new system of free open access publications.

- Study neuroscience.

What should we do about philosophy of mathematics?

The following informal concepts of mathematical practice cry out to be explicated:

beautiful, natural, deep, trivial, "right", difficult, genuinely, explanatory . . .

– Timothy Gowers

Much more is in my blog and book:

Mathematics under the Microscope

Available for free from the Internet:

Google for "Mathematics under the Microscope"