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INCLUDING SUPPRESSION EFFECTIVENESS IN FIRELINE GROWTH MODELS

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Abstract
The inclusion of suppression effectiveness in fire line growth models is formulated as a system of differential equations. The model draws on earlier ideas using ellipses to model fire growth, particularly the head fire and flank fire rates of spread, combines this with recent studies of the effect of fire line on spread rate and appends a single equation for the increase of suppressed fire line with time. Representative parameter values are used to illustrate this way of describing the effect of fire suppression activities on the fire line and to develop criteria for the likely outcome of containment activities.

Introduction
Wildland fire spread modelling has been an active research topic for many years and is the subject of a recent review by Pastor et al (2003). Including chemical kinetics, such as in the paper by Assensio and Ferragut (2002), and the effects of suppression on fire growth and the eventual fire size; for example, Anderson (1989), are some of the options previously considered in order to make the models more realistic.

In the present paper, we introduce a new option for modelling fireline growth when there is active suppression applied from a set time after the fire was initiated. In particular, we use ideas from ellipse modelling of fire growth to provide the head fire and the flanking fire rates of spread. This is combined with information about the effect of fireline length on these spread rates (Cheney and Sullivan (1997)) and also linked with the rate of fireline growth in the presence of suppression. We use the mathematical framework of dynamical systems to illustrate this way of describing the effect of fire suppression activities on the fireline and to develop criteria for the likely outcome of containment activities.

It is anticipated that each component of the present model could be refined to account for more details of actual situations and that the result could then form a simulation module within fire incident management systems.

Fireline Growth Model
Let our fireline at any time t have a total length $F(t)$. The requirement of a point ignition gives the initial condition that $F(0) = 0$. The total fireline should increase in length for all $t > 0$; at least until the fire is extinguished. We also formally divide the fireline into two parts: an active part $L(t)$ and a suppressed part $S(t)$ as illustrated in Figure 1.

Inspired by the ellipse models (Anderson et al (1982)) and following on from Weber and Sidhu (2005), we write an equation for fireline growth as:

$$\frac{dL}{dt} = \alpha(\frac{da}{dt} + \frac{db}{dt}) - \frac{dS}{dt} \tag{1}$$

Here α is a geometric constant with value between 1 and 2 (its precise value in any given fire will depend on the fire shape), $\frac{da}{dt}$ represents the head fire rate of spread, $\frac{db}{dt}$ represents the flanking fire rate of spread and $\frac{dS}{dt}$ accounts for the conversion of active fireline into suppressed fireline through suppression activities.

To be able to solve this model, we need several other pieces of information. From the literature; e.g. Cheney and Sullivan (1997), we deduce that the rate of forward spread depends upon the head fire width (for any given wind speed) in a way that can be described by the relation

$$\frac{da}{dt} = ROS_h(1 - e^{-\beta L}) \tag{2}$$

where ROS_h is the potential rate of head fire spread under the prevailing conditions and β is a constant.

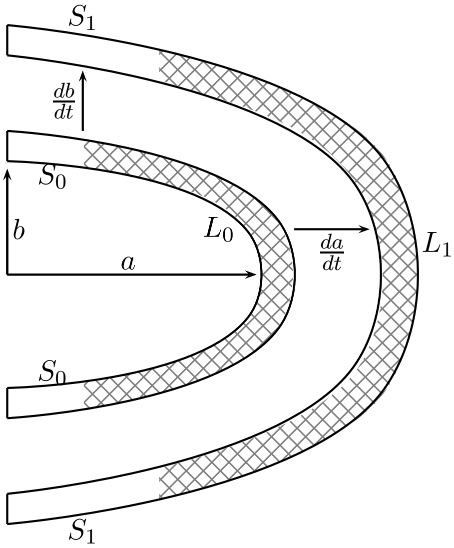

Figure 1   Schematic of a Partly Suppressed Fireline

We also assume that the flanking rate of spread, ROS_f, (typically a significantly smaller number than the head fire rate of spread) is described be a similar equation:

$$\frac{db}{dt} = ROS_f(1 - e^{-\beta L}) \tag{3}$$

Note that the ratio ROS_h/ROS_f can be estimated from the length to breadth ratios often quoted for fires.

The rate of suppression is expressed in terms of available resources through

$$\frac{dS}{dt} = Q(t) \tag{4}$$

and this can be used to allow us to arrive at the final model

$$\frac{dL}{dt} = \alpha(ROS_h + ROS_f)(1 - e^{-\beta L}) - Q(t) \tag{5}$$

subject to the initial condition $L(0) = L_0$ and which we must solve for $L(t)$.

We note that it is not possible to write a closed form solution for $L(t)$ for this model. Nevertheless it is reasonably simple to write a program to numerically determine the solution at any time, as we shall discuss later. First we shall discuss possible parameter values and examine some of the limiting cases where good approximate solutions can be found.

Parameter Values

To be able to use this model, we need to determine suitable values for each of the parameters. Clearly the values will depend upon the particular fires that might be considered, but for the purposes of illustration, we shall select as representative values $ROS_h = 4,000m/hr$ and $ROS_f = 1,000m/hr$ and use these throughout this paper.

The parameter α will depend upon the geometry of the changing fireline and will probably vary from around $\alpha = 1.4$ for typical cases to $\alpha = 2$ as a likely maximum for extreme fire behaviour. This latter value can be used to obtain conservative estimates, as in the next section.

The time it takes the fireline to be sufficiently large to have reached its maximum potential spread rate determines the parameter β and examining the curves in Cheney and Sullivan (1997), we estimate $\beta = 0.03$ to be a typical value for moderate fires.

Values for the available suppression resources Q can be estimated for a variety of active suppression methods. A useful guide is McCarthy et al (2003), from which we deduce that numbers such as $700m/hr$ are possible by a single dozer and significantly larger values are possible if several appliances and methods can be combined.

The initial fireline length L_0 can vary enormously and we will use several possible values in this paper to illustrate the potential application of the model.

Large Fireline Length
For large fireline length, $L(t)$, the head fire and flank fire rates of spread reach their full potential values. Then the non-linear exponential term becomes extremely small. If we also have constant available suppression resources (Q independent of time), then in this case the solution is very well approximated by

$$L(t) = L_0 + (\alpha(ROS_h + ROS_f) - Q)t \tag{6}$$

For the suppression activities to be successful in this limit, we require

$$Q > \alpha(ROS_h + ROS_f) \tag{7}$$

and we can estimate the suppression time required by

$$t_{sup} = L_0/(Q - \alpha(ROS_h + ROS_f)) \tag{8}$$

To illustrate this, we use the values $ROS_h = 4,000m/hr$ and $ROS_f = 1,000m/hr$ as discussed in the previous section, we set $\alpha = 2$ to be representative of extreme conditions, we assume quite active suppression such as $Q = 12,000m/hr$ and we begin with an initial fireline of length $L_0 = 4,000m$.

As the available suppression resources satisfies equation (7), we can be confident that in this case the suppression activities will be successful and we can estimate from the equation for t_sup that it will take approximately two hours to contain this fire.

Note that the time scales linearly, so that if the initial fireline is twice as long, then the suppression time is also twice as long.

Very Small Fireline Length
For very small fireline length, the head fire and flank fire rates of spread are very small, so that suppression is the dominating effect and the approximate solution to our model (again assuming constant Q) in this case is

$$L(t) = L_0 - Qt \tag{9}$$

reflecting the well known fact that every fire can be extinguished provided suppression arrives sufficiently early. From this we can also obtain an estimate of the "early extinction time", t_{ee} as

$$t_{ee} = L_0/Q \tag{10}$$

For example, if $L_0 = 10m$ and $Q = 20m/hr$, then $t_{ee} = 30$ minutes.

General Criteria and Estimates
In general, we are interested in the application of the model to situations where there is a dynamical interaction between the fire growth and the fire suppression activities and we are particularly interested in using the model to obtain criteria for the likely success and estimates of the time it will take for successful suppression.

To this end, in this section we analyse the model with as few assumptions as possible. We begin by noting that the exponential $1 - e^{-\beta L}$ is always less than unity for any fireline length L. Hence, provided that suppression activities are maintained so that

$$\frac{dS}{dt} > \alpha(ROS_h + ROS_f) \tag{11}$$

fire suppression will always succeed no matter how large the initial fireline length is at the commencement of suppression activities. This is really just a common sense conclusion and as a conservative estimate to guarantee success we would also let $\alpha = 2$ and then restate our criteria in words as

- calculate potential head fire and flank fire rates of spread
- add these and multiply by two
- maintain the rate of construction of suppressed fireline so that is greater than this last number

An example calculation illustrating this is as already done in section 5 and we note that this shows that treating every fireline as a potentially extreme fire will provide conservative estimates for the resources required for successful suppression.

While this guarantees success, there is still the possibility of suppression success with less suppression activity. For example, we can maintain decreasing L (i.e. keep $\frac{dL}{dt} < 0$) at all times provided

$$\alpha(ROS_h + ROS_f)(1 - e^{-\beta L}) < Q(t) \tag{12}$$

If we apply this to the initial fireline length L_0, assume constant Q and rearrange, we obtain

$$L_0 < -\frac{1}{\beta}\ln(1 - \frac{Q}{\alpha(ROS_h + ROS_f)}) \tag{13}$$

as an estimate for the maximum initial fireline length which can be suppressed with resources Q. Note that this estimate can only be applied provided that the inequality $Q < \alpha(ROS_h + ROS_f)$ is satisfied. This means that this estimate must be applied with care. An example is the parameter values used in section 5 but with the significantly smaller value for the suppression resources: $Q = 9,000m/hr$. Then we obtain a maximum initial fireline length of only $76m$ (rounded down) for suppression to be successful.

We could also estimate the time it will take to suppress any such initial fireline with L_0 less than this with the equation

$$t = \frac{L_0}{Q - \alpha(ROS_h + ROS_f)(1 - e^{-\beta L_0})} \tag{14}$$

which gives a rather small time of $0.04hr$. In fact, these approximations are not uniformly valid as L decreases from a value like 70 to zero and it is typically much easier, safer and accurate to simply solve the differential equation numerically as will be done in the next section.

Contrasting these findings shows that, for this example, suppression resources of $12,000m/hr$ would be able to succeed against any initial fireline, whereas suppression resources of $9,000m/hr$ would only be able to succeed provided that the initial fireline length is less than the rather modest value of $76m$ (note that numerical solution techniques as discussed in the next section will revise this value to the more accurate $69m$ which indicates that there is a small but potentially important level of error with the approximations that result in equations 13 and 14).

Numerical Solution

The model presented in this paper can be easily solved numerically using a variety of routines and software applications. Possibly the simplest is the Euler method where the derivatives are approximated by differences and used to update the solution in small time steps. Provided the time steps are small enough the error in this method is also kept reasonably small, the results are obtained very quickly and the method can be easily programmed with any software such as fortran, basic, matlab or excel.

The essence of the method applied to our model can be stated as follows. Begin with an initial fireline length L_0 and update it in small time steps Δt according to

$$L_{new} = L_{old} + (\alpha(ROS_h + ROS_f)(1 - e^{-\beta L_{old}}) - Q(t))\Delta t \tag{15}$$

We also note that this numerical method easily allows us to calculate the solution even when the suppression resources, $Q(t)$, vary with time; an obvious improvement on our previous analytical approximations.

To illustrate the numerical solution of the full model, we select the values for the model: $\alpha = 1.4$, $\beta = 0.03$, $ROS_h = 4,000m/hr$, $ROS_f = 1,000m/hr$ and the rather modest initial fire line length of $L_0 = 1,000m$. Then we can immediately see the consequences of varying the available suppression resources Q. If Q is less than $7,000m/hr$, the fire will continue to grow and the suppression activity will never be able to contain the fire.

Conclusion

To recap the essential features of our model and the solutions we wish to highlight the most salient results in the present paper.

To establish a safe, conservative estimate for successful suppression, we can refer to equation 7 which allows us to calculate a level of suppression activity which will ensure success. Then we can also estimate the time for which this level of suppression needs to be maintained for any initial fireline length using equation 8. It should be emphasised that this is most likely to be the usual situation and that equations 7 and 8, with suitable field testing, are most likely to be the most useful simple results from the present model.

On the other hand, if we wish to adopt a strategy of early intervention, then (as long as the initial fireline length is sufficiently small as estimated by $L_0 < \frac{1}{\beta}$), equation 10 can be applied to determine the time required for suppression.

The numerical solution presented here provides a way of simulating the growth of the fireline with any desired suppression strategy. With suitable field verification and calibration this could be used to predict the likely failure or success depending upon the level of applied suppression resources and the size of the fireline when the suppression activities first commence.

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