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Onset of Flow-Induced Fingering in Bushfires

John Dold[1], Gregory Sivashinsky[2], Rodney Weber[3]

[1]School of Mathematics, University of Manchester, M60 1QD, UNITED KINGDOM

[2]School of Mathematical Sciences, Tel-Aviv University, Tel-Aviv 69978, ISRAEL

[3]School of Physical, Environmental and Mathematical Sciences, UNSW at ADFA, Canberra, ACT 2600, AUSTRALIA
(also: Bushfire CRC, Level 5, 340 Albert Street, East Melbourne, Victoria 3002, AUSTRALIA)

Abstract

On a relatively large length-scale, the fire-fronts of wind-driven bushfires are sometimes seen to develop into curved shapes, suggesting that a linear fire-front becomes unstable. A mechanism for this instability can be identified if the hot plume of the fire is considered to partially block the air-flow from below, while stratification of the atmosphere restricts upward displacement. Downwind of the fire this causes a speeding up of the component of the average horizontal flow in the direction normal to any part of the fire-front. The perturbation in the horizontal wind that results from a perturbed shape of the fire-front shows an increase in the flow of air into the fire at more advanced parts of the front, normally resulting in an increased burning rate which would therefore increase the size of the perturbation.

1 Introduction

Fires in the landscape (bushfires) are a common phenomenon in many parts of the world during drier months of each year. Significant progress has been made over many years in the understanding of the behaviour and ecological impacts of these fires, but there remain several outstanding issues relating to actual observations (of both planned and unplanned fires) which are not well understood.

A recent review by Pastor et al [5] gives a good account of what has been achieved and the way in which current understanding is being incorporated into technical aides for landscape fire agencies, but it also notes that there are significant gaps in our understanding of landscape fire dynamics. The paper by Asensio and Ferragut [2] is an example of the numerical methods required when including a description of the chemical kinetics of the combustion, but this still leaves many issues of the dynamics and stability of landscape fire fronts unanswered.

Among these are the tendency for landscape fires to form a curved front after any type of ignition and then to propagate in a steady fashion, maintaining the curved front shape. Such a shape may arise as a development from an instability in a linear fire front [3]. In this article we show how an instability of this kind can arise under suitable atmospheric conditions.

2 Flow around a bushfire

2.1 Fire-front and burning rate

Consider a bushfire in a relatively thin layer of uniform vegetation on a horizontal terrain, with the burning taking place along a narrow front that follows a continuous path parameterised by $\mathbf{r} = \mathbf{R}(t,s)$, where $\mathbf{r} = (x,y,z)$ is a spatial coordinate, t is time and s is the arclength, measuring distance along the fire-front. The definition of s can be chosen such that unburnt material lies to the left and burnt residue of the fire lies to the right when facing in the direction of increasing s. Tangent and normal unit vectors can then be identified as

$$\widehat{\mathbf{s}} = \mathbf{R}_s, \qquad \widehat{\mathbf{n}} = \widehat{\mathbf{k}} \times \widehat{\mathbf{s}} \tag{1}$$

so that the normal direction $\widehat{\mathbf{n}}$ is rotated about the vertical direction $\widehat{\mathbf{k}} = (0,0,1)$ by $\pi/2$ from the tangential direction $\widehat{\mathbf{s}}$. A burning rate at a point $\mathbf{R}(t,s)$ can then be defined as

$$\mu(t,s) = \mathbf{R}_t \cdot \widehat{\mathbf{n}} \tag{2}$$

representing the rate of advancement of the interface into the unburnt material. If the vegetation is completely burnt in the fire, or at least a fixed fraction of it, then energy is released at a rate that is proportional to μ, per unit length of the fire-front.

How the fire-front propagates must be influenced by the rate of air flow into the fire in the immediate vicinity of the foot of the fire where the edge of the burning vegetation is to be found. This paper aims to provide an idealised, but reasonable, estimate of the effect of the fire itself on this airflow into the fire, offering a possible feedback mechanism for the development of a straight line of fire into a more curved shape.

2.2 Effect of the fire-plume on atmospheric wind

The plume produced by the fire can be considered to present some blockage to any incoming wind, as illustrated in Figure 1. In the near vicinity of the fire, it can be thought of as rising to some height (which may depend on the burning rate μ and the velocity of the incoming wind) and thereafter confining the wind to pass above this height. In a stably stratified atmosphere, the average air-flow is therefore forced into a narrower layer so that it must also increase in speed. On a scale that is large compared with the height of the plume the flow can be considered to undergo a fairly abrupt change as it passes over the fire.

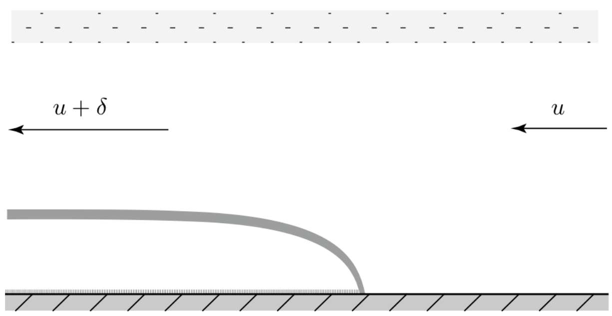

Figure 1: An illustration of the blocking effect of a fire-plume with vertical displacement of the flow restricted by a stably stratified atmosphere. The horizontal wind speed increases by an amount δ from its incoming value of u. The fire is taken to be moving into unburnt vegetation on the left.

The speed of advancement of the fire-front is generally much slower than the wind-speed, so that the air-flow can be considered to pass over an almost stationary fire. Thus, if $\mathbf{u}$ denotes the average horizontal wind-speed ahead of or above any plume, it can be taken to satisfy the steady incompressible Euler equations

$$\nabla\cdot\mathbf{u}=0, \quad \mathbf{u}\cdot\nabla\omega=0, \quad \omega=\nabla\times\mathbf{u} \tag{3}$$

where ω is the vorticity. Writing $\mathbf{u}=(u,v)$ where u is the velocity in the x direction and v is in the y direction, the Euler equations can be written as

$$u_x+v_y=0, \quad u(v_{xx}+v_{yy})=v(u_{xx}+u_{yy}). \tag{4}$$

These equations will be used to describe the average horizontal flow velocity $\mathbf{u}$ on either side of the fire, but not in the immediate vicinity of the fire where fairly abrupt changes in the flow must occur, including significant vertical motion. The most simple way of modelling these changes is to apply jump conditions across the path $\mathbf{r}=\mathbf{R}$ in the form

$$[\![\mathbf{u}]\!]=\delta\widehat{\mathbf{n}} \tag{5}$$

where $[\![\cdot]\!]$ denotes the value on the unburnt side, of the quantity between the double brackets, minus the value on the burnt side. Thus the quantity δ represents an increase in velocity in the normal direction $\widehat{\mathbf{n}}$, as sketched in Figure 1. Continuity of the flow requires that the tangential component of the velocity should not change.

On smaller length-scales, of the order of the height of the plume, changes in the average horizontal wind-speed will not be abrupt. Results based on using (4) and (5) to calculate the local wind-flow into the fire must be interpreted in this light, as discussed later.

2.3 Burning rate, blockage and wind-speed

If we suppose that a perfectly linear infinitely long fire-front is subjected to a wind-speed $\bar{v}_n$, blowing into it in the normal direction from the burnt vegetation, then the burning rate μ and the degree of increase in wind-speed δ should both depend, in some way, on $\bar{v}_n$. Field observations and experiments [4] have led to relationships of the form $\mu=\mu(\bar{v}_n)$, invariably with the burning rate being an increasing function of the wind-speed.

For given atmospheric conditions, in which (say) a thermocline at some altitude above the fire prevents vertical displacement, with neutral stratification below that level, one would expect the height to which the plume rises to depend on the dimensionless ratio of burning rate to incoming flow speed, $\mu/\bar{v}_n$. The degree of blockage should also depend on this dimensionless ratio and, for a fixed degree of blockage, the jump in velocity should then be proportional to the incoming velocity itself.

Accordingly, we can propose the general type of formulae

$$\mu=\mu(\bar{v}_n) \quad \text{and} \quad \delta=\bar{v}_n\,\sigma(\mu/\bar{v}_n) \tag{6}$$

for some function $\sigma(\cdot)$, to describe the dependence of μ and δ on the incoming wind $\bar{v}_n$. Because the degree of blockage should increase as the ratio $\mu/\bar{v}_n$ increases, both of the functions $\mu(\cdot)$ and $\sigma(\cdot)$ are likely to be increasing functions with positive first derivative. These can be taken to be given functions in this study although, of course, their actual dependence on wind-speed is a much deeper question involving the nature of the vegetation, moisture content, heat transfer, chemical behaviour, energetics of the burning undergrowth and atmospheric conditions at the time.

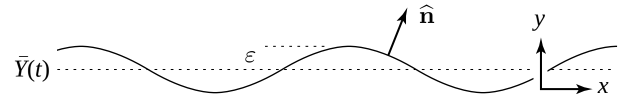

Figure 2: Sketch of an oscillatory fire-front with amplitude of disturbance ε.

3 Stability of a linear fire-front

If axes are chosen so that the x-axis lies parallel to a linear fire-front then the fire can be considered to follow the path $y\equiv\bar{Y}(t)=\mu t$. With an incoming wind such that $\mathbf{u}\to(\bar{u},\bar{v})$ as $y\to-\infty$, the Euler equations and the jump conditions give rise to the flow-field

$$\mathbf{u}=\begin{cases}(\bar{u},\bar{v}+\delta) & \text{for } y>\bar{Y}\\ (\bar{u},\bar{v}) & \text{for } y<\bar{Y}\end{cases} \tag{7}$$

in which the normal flow velocity increases by δ on crossing the front. Such a front may be unstable to small perturbations.

3.1 Flow around a nearly linear fire-front

If, at some moment, the fire is located on the path $y=\bar{Y}+\varepsilon e^{ikx}$, where $\varepsilon(t)$ is a small amplitude of displacement with wavenumber k about the mean location $\bar{Y}(t)$, the normal direction can be written as

$$\widehat{\mathbf{n}}=(-\varepsilon ike^{ikx},1)+\mathrm{O}(\varepsilon^2) \tag{8}$$

showing only the horizontal components of the vector. The rate of advancement can be written as

$$\mu=\mathbf{R}_t\cdot\widehat{\mathbf{n}}=\bar{Y}_t+\varepsilon_t e^{ikx}+\mathrm{O}(\varepsilon^2). \tag{9}$$

The wind flow $\mathbf{u}$ will be disturbed by an amount of order ε which can be determined in the limit as $\varepsilon\to0$.

It is useful to write the wind velocity as an asymptotic expansion, for $\varepsilon\ll1$, in the form

$$\mathbf{u}=(u_0,v_0)+\varepsilon e^{ikx}(u_1,v_1)+\mathrm{O}(\varepsilon^2) \tag{10}$$

in which u_0 and v_0 should be piecewise constant, as in (7). By defining $\eta=y-\bar{Y}$, the dependence of u_1 and v_1 on y can be examined in the form

$$(u_1,v_1)=(a,b)e^{\lambda\eta} \tag{11}$$

where a and b are constant. Equations (4) then give

$$ika+\lambda b=0, \quad (\lambda^2-k^2)(v_0\lambda+iku_0)=0 \tag{12}$$

for which there are three eigenvalues given by $\lambda=\pm|k|$, representing irrotational disturbances in the incompressible flow, and $\lambda=-iku_0/v_0$ which represents the advection of any vorticity with wavenumber k that may be generated at the fire-front. Because the horizontal flow-field must be bounded at infinity, the perturbation to the wind-velocity can now be written as

$$(u_1,v_1)=\begin{cases}(|k|,ik)Ae^{-|k|\eta}+(u_0,v_0)Ce^{-iku_0\eta/v_0} & : \ \eta>0\\ (|k|,-ik)Be^{|k|\eta} & : \ \eta<0\end{cases}$$

for constants A, B and C. However it can be noted that the flow of air is considered to pass over the interface; there is no mechanism for the production of vorticity and so we may immediately infer that $C=0$.

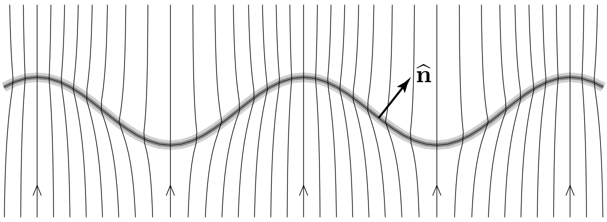

Figure 3: Illustration of the deflection of the streamlines around a sinusoidally shaped fire-front. The horizontal air-flow into the fire is diminished at rearward parts of the front but increased at more advanced parts.

Using a Taylor expansion of the solutions for $\eta > 0$ and $\eta < 0$ the jump condition (5) can be written in terms of a jump condition applied at $\eta = 0$ in the form

$$\begin{aligned}[\![(u_0, v_0) + (u_0' + u_1, v_0' + v_1)\varepsilon e^{ikx}]\!] \\ = (\delta_0 + \varepsilon e^{ikx}\delta_1)(-\varepsilon ike^{ikx}, 1) + \mathrm{O}(\varepsilon^2)\end{aligned} \tag{13}$$

where the primes denote differentiation with respect to η and in which δ has been expanded to $\delta_0 + \varepsilon e^{ikx}\delta_1$. It follows that

$$\begin{aligned}[\![u_0]\!] &= 0 & [\![v_0]\!] &= \delta_0 \\ [\![u_0' + u_1]\!] &= -ik\delta_0 & [\![v_0' + v_1]\!] &= \delta_1.\end{aligned} \tag{14}$$

The remaining conditions are given, as before, by the far-field limit

$$\lim_{\eta \to -\infty} \mathbf{u} = (\bar{u}, \bar{v}). \tag{15}$$

This leads to the solutions, for $\eta > 0$ and $\eta < 0$ respectively:

$$\mathbf{u} = \begin{cases} (\bar{u}, \bar{v} + \delta_0) + \varepsilon e^{ikx}\left(\dfrac{-i}{2k}, \dfrac{1}{2|k|}\right)(k^2\delta_0 + \delta_1|k|)e^{-|k|\eta} \\ (\bar{u}, \bar{v}) + \varepsilon e^{ikx}\left(\dfrac{i}{2k}, \dfrac{1}{2|k|}\right)(k^2\delta_0 - \delta_1|k|)e^{|k|\eta} \end{cases}$$

after ignoring contributions of order ε^2. Flow-lines predicted by this solution are illustrated in Figure 3, showing how an increase in air-flow into the fire arises at more advanced parts of the fire-front. In particular, this formula suggests that the normal flow coming into the fire-front at $\eta \approx \varepsilon e^{ikx}$ is, to order ε

$$u_n = \mathbf{u}^- \cdot \widehat{\mathbf{n}} = \bar{v} + \varepsilon e^{ikx}\left(\tfrac{1}{2}(|k|\delta_0 - \delta_1) - ik\bar{u}\right) \tag{16}$$

which demonstrates an increase in wind-speed in regions where $\varepsilon e^{ikx} > 0$, provided δ_1 is small enough, and a phase shift caused by transverse flow.

3.2 Feedback from the wind-flow

Some careful interpretation is now needed because the model (4) and (5) for the wind-flow is not valid very close to the fire-front where velocity has a significant vertical component and advection into the fire-plume is important.

Instead, we may consider some distance h upwind of the fire-front where the model is more likely to hold; this distance should be comparable with the height to which the plume rises. At $\eta \approx -h + \varepsilon e^{ikx}$ the horizontal normal velocity is

$$u_{nh} = \bar{v} + \varepsilon e^{ikx - |k|h}\left(\tfrac{1}{2}(|k|\delta_0 - \delta_1) - ik\bar{u}\right) \tag{17}$$

which has the same structure as (16) and gives almost the same value at small wavenumbers but which is decreased substantially at large wavenumbers (with wavelengths of the order of the plume height).

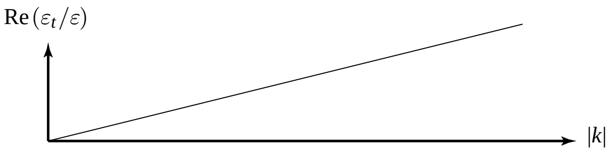

Figure 4: Linear dependence of the growth-rate of an instability $\mathrm{Re}(\varepsilon_t/\varepsilon)$ with the wavenumber $|k|$, for small wavenumbers, provided $\delta_0' > -2$.

It is reasonable now to consider the burning rate μ to depend on u_{nh}. A more detailed examination of the flow in the region around the fire and its plume, where (4) and (5) are not valid, would have the velocity u_{nh} as an incoming flow. The wind flow feeding air into the foot of the fire would then be proportional to u_{nh}, at least for small wavenumbers k. At larger wavenumbers stronger three-dimensional effects are likely to reduce the contribution of the perturbation below that suggested by (17) although the trend is still likely to be the same.

At wavelengths of the order of the height of the plume, or shorter, there are likely to be other stabilising effects not examined here. These may include transverse heat transfer through radiation and entrainment of air into the plume. However, to demonstrate a potential source of instability it is necessary only to consider long wavelengths ($|k|h \ll 1$) for which u_n can be used in place of u_{nh}. The effects of the instability should certainly be diminished at shorter wavelengths. Using the formulae (6) we can suppose that

$$\mu = \mu(u_n) \quad \text{and} \quad \delta = u_n\sigma(\mu/u_n). \tag{18}$$

at sufficiently long wavelengths, with $|k|h \ll 1$.

3.3 Conditions for instability

Substituting the formula (16) into (18) leads to the expression

$$\begin{aligned}\mu = \mathbf{R}_t \cdot \widehat{\mathbf{n}} &= \bar{Y}_t + \varepsilon_t e^{ikx} \\ &= \mu(\bar{v}) + \varepsilon e^{ikx}\left(\tfrac{1}{2}(|k|\delta_0 - \delta_1) - ik\bar{u}\right)\mu'(\bar{v})\end{aligned} \tag{19}$$

so that

$$\bar{Y}_t = \mu(\bar{v}) \quad \text{and} \quad \varepsilon_t = \varepsilon\left(\tfrac{1}{2}(|k|\delta_0 - \delta_1) - ik\bar{u}\right)\mu'(\bar{v}) \tag{20}$$

in which (after some algebraic manipulation) we can set

$$\begin{aligned}\delta_0 = \bar{v}\sigma_0 \quad &\text{and} \quad \delta_1 = \frac{|k|\delta_0 - 2ik\bar{u}}{2 + \delta_0'}\delta_0' \\ \delta_0' = \left.\frac{\mathrm{d}\delta}{\mathrm{d}u_n}\right|_{u_n = \bar{v}} &= \sigma_0 + \frac{\mu_0'\bar{v} - \mu_0}{\bar{v}}\sigma_0'\end{aligned} \tag{21}$$

where $\mu_0 = \mu(\bar{v})$, $\mu_0' = \mu'(\bar{v})$, $\sigma_0 = \sigma(\mu_0/\bar{v})$ and $\sigma_0' = \sigma'(\mu_0/\bar{v})$. As shown in (19), because the burning-rate μ gives the rate of advancement of any front, the formula for μ can be used to determine whether or not a perturbation in the shape of the fire-front grows in size. This is made explicit in equation (20).

Taking $\mu'(\bar{v})$ to be positive, as it should be, equation (20) shows that the growth-rate of a perturbation of the form εe^{ikx} depends on the size of the real part of $\left(\tfrac{1}{2}(|k|\delta_0 - \delta_1) - ik\bar{u}\right)$. The imaginary part contributes only to the rate of transverse displacement of the perturbation. Noting that

$$\mathrm{Re}\,\tfrac{1}{2}(|k|\delta_0 - \delta_1) = \frac{|k|\delta_0}{2 + \delta_0'} \tag{22}$$

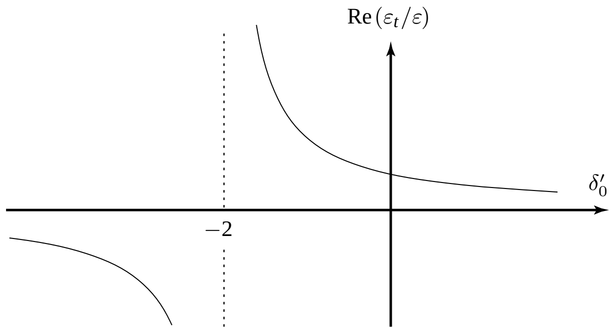

Figure 5: Dependence of the growth-rate on δ_0', which represents the rate of change of the flow increase δ with incoming wind-speed, at a fixed wavenumber k. The fire-front becomes stable if $\delta_0' < -2$.

it can be seen that the growth (or decay) rate is proportional to $|k|$ at long wavelengths, as sketched in Figure 4. Generally speaking, we would expect δ_0' to be positive, meaning that the jump in wind speed δ across the fire-front increases as the wind-speed increases.

However it is interesting to note that, if δ decreases (but not too rapidly) as u_n increases, the growth-rate of the instability is actually enhanced, as shown in Figure 5 at a fixed wavenumber k, creating a very rapid growth-rate if δ_0' approaches -2 (provided $\delta_0' > -2$). Under such unstable conditions, less advanced parts of the fire-front would tend to block the flow more from behind, channeling even stronger wind-flow towards more advanced parts of the front.

On the other hand if δ decreases sufficiently rapidly, as u_n increases, to make $\delta_0' < -2$ then the effect dramatically reverses, completely stabilising a linear fire-front.

4 Discussion

The notion of blockage of the wind-flow into a fire-front in a stably stratified atmosphere, as a result of the plume created by the fire itself offers a relatively simple mechanism for generating an instability in the shape of the fire-front. A fuller evolution of the fire-front could then lead towards the propagation of the front preferentially as fingers of burning.

However, it must be noted that the model employed here for the blockage might not be adequate in all circumstances. The formulae (6) may capture the essence of blockage by a plume for perfectly linear fronts and if such formulae were to still apply to perturbed fronts, as assumed in (18), it is very likely that one would find $\delta_0' > 0$, a condition for instability at large enough wavelengths.

But a plume consists of light hot gas rising through more dense colder gases, possibly rising at an angle because of the wind. Under such conditions the Rayleigh-Taylor instability would operate tending to enhance the upward movement of the plume in regions that rise first. This would have an effect similar to cases in which $\delta_0' < 0$. Ironically, as we have found, this may serve either to enhance the instability or to stabilise a linear fire-front under suitable conditions.

The exact nature of the full form of this interaction would need further study, although the simple notion of blockage by a fire-plume in a stably stratified atmosphere is relatively easily examined. Instability is predicted if the overall jump in wind-speed across the front grows with increasing incoming air-flow at large enough wavenumbers. The instability is even present if the jump in wind-speed decreases, provided the decrease is not too rapid.

5 Acknowledgements

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