Geometric Mechanics and Symmetry: The Peyresq Lectures

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Geometric Mechanics and Symmetry

The Peyresq Lectures

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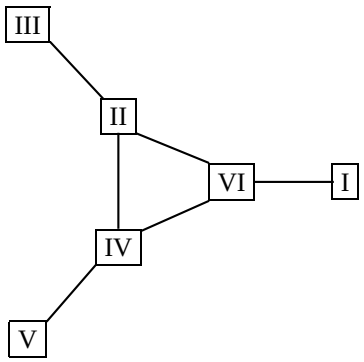
Each set of lectures begins with its own table of contents.



Interrelationships between the courses.

Preface

In the summers of 2000 and 2001, we organized two European Summer Schools in Geometric Mechanics. They were both held in the wonderful environment provided by the village-cum-international conference centre at Peyresq in the Alpes de Haute Provence in France, about 100km North of Nice. Each school consisted of 6 short lecture courses, as well as numerous short talks given by participants, of whom there were about 40 at each school. The majority of participants were from Europe with a few coming from West of the Atlantic or East of the Urals, and we were pleased to see a number of participants from the first year returning in the second. Several of the courses and short talks led to collaborations between participants and/or lecturers.

The summer schools were funded principally by the European Commission under the High-Level Scientific Conferences section of the Fifth Framework Programme. Additional funding was very kindly provided by the *Fondation Peiresc*. The principal aim of the two schools was to provide young scientists with a quick introduction to the geometry and dynamics involved in geometric mechanics and to bring them to a level of understanding where they could begin work on research problems. The schools were also closely linked to the Mechanics and Symmetry in Europe (MASIE) research training network, organized by Mark Roberts, and several of the participants went on to become successful PhD students or postdocs in MASIE.

Of the lecture courses, seven have been written up for this book—mostly by the participants themselves with varying degrees of collaboration from the lecturers. The book is divided into 6 chapters, each representing a course of 5 or 6 lectures, with the exception of Ratiu's which are taken from two courses. The notes on Stability in Hamiltonian systems by Rink and Tuwankotta based on Meyer's lectures on N-body problems have been placed first as they require the least background knowledge. They cover not only Lyapounov's and Dirichlet's stability theorems but also the *in*stability theorem of Chetaev, with applications to the restricted 3-body problem. Second are the notes from Ratiu's courses which give an introduction to the mathematical formalism of geometric mechanics, beginning with the Hamiltonian, Lagrangian and Poisson formalisms, and continuing with aspects of reduction and reconstruction, the whole being laced with numerous examples, and including some material on Euler-Poincaré equations. This last topic is the basis of the third set of lecture notes: Holm's course on the Euler-Poincaré approach to fluid dynamics, showing especially how this approach helps to model the multiscale physics involved.

The fourth chapter contains Cushman's lectures on the global geometry of integrable systems, describing particularly the monodromy in such systems,

which has recently proved to be so important in explaining some features of molecular spectra. When integrability breaks down, one requires KAM theory which is described in Broer's lectures presented in the following chapter. The theory is described there for dissipative systems, showing how quasiperiodic attractors persist and bifurcate in families of systems, but applies also to conservative systems as is described in the appendix to that chapter.

The final chapter consists of (a slightly expanded version of) Montaldi's lecture course on Hamiltonian bifurcations in symmetric systems. These deal firstly with bifurcations near equilibria including Hamiltonian-Hopf bifurcation, and then with bifurcations of relative equilibria.

We believe all the participants and lecturers would like to join us in thanking Mme. Mady Smets and the staff of the *Peyresq Foyer d'Humanisme* for their warmth, generosity and hospitality, and for the smooth running of the centre without which the Schools would not have had the academic success they did.

James Montaldi, Tudor Ratiu.
Summer, 2004

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