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Conductivity perturbations in EIT

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Abstract: Difference EIT reconstructs a signal generated by conductivity contrasting regions. We explore the effect that, for large contrasts, conductive regions produce a greater signal than non-conductive ones, and this difference is determined by the region shape.

1 Introduction

EIT images conductivity contrasting regions within a body. Image reconstruction algorithms (especially for difference imaging) are based on linearization of the sensitivity matrix. For this reason, the sensitivity of EIT to small contrasts is well understood. A small change of $\sigma+\Delta\sigma$ produces the same magnitude of signal change as $\sigma-\Delta\sigma$ (or, in logarithmic space, $\sigma^{1+\Delta}$ and $\sigma^{1-\Delta}$). However, in practice, conductivity contrasts can be large (i.e. air in the lungs, or hypertonic saline in the blood).

In this paper we seek to understand the strength of EIT signals as a function of the conductivity contrast ratio. Our analysis is based on a simplification that the target is far from measuring electrodes relative to its size (which is reasonable for many biomedical applications of EIT). In this case, the contrast can be understood in terms of the polarization tensor, for which there is a well developed theory[1].

2 Perturbations and current streamlines

Perhaps surprisingly, the most important factor is the shape of the target area with respect to the current stream lines. A non-conductive target has most effect when it is mostly perpendicular to the streamlines, while, for a conductive target, the effect is largest when parallel.

In order to illustrate this effect, we simluate an elliptical region of conductivity, σ, in an otherwise homogeneous rectangular region of unity conductivity. Two electrodes are placed at each horizontal end to produce a horizontal voltage gradient. As shown in Fig.1, for a non-conductive contrast (top row, with $\sigma\approx.05$), the most significant effect is when the major axis of the ellipse is oriented perpendicular to the streamlines. On the other hand, for a conductive contrast (bottom row, with $\sigma\approx20$), the most significant effect is the converse, when the major axis is parallel to the streamlines.

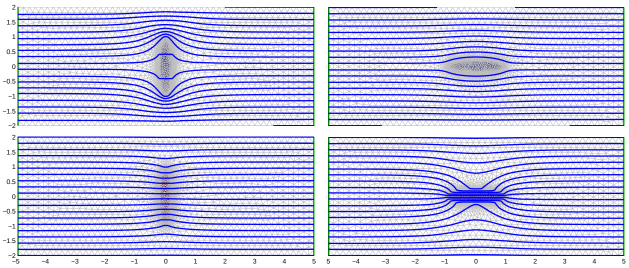

Figure 1: Streamlines surrounding an elliptical inclusion of various contrasts in 2D. The elliptical horizontal/vertical major axis dimension is *left column*: 2.0 and *right column*: 0.5. The contrast/background ratio was *top row (non-conductive)*: $\exp(-3.0)\approx .05$ and *bottom row (conductive)*: $\exp(+3.0)\approx 20$.

This makes intuitive sense. A contrasting region which which blocks the streamlines will have the most effect when it is non-conductive. However, a region which follows close to the streamlines will, if conductive, draw in the streamlines, but, if non-conductive, have little effect.

3 Conductivity contrasts and EIT

In order to understand the how this effect manifests itself into an EIT system in 3D, we consider a simplified chest model as a cylinder with 16 electrodes in a central plane (with hight/diameter ratio of one). A small central ellipsoidal region is created as a conductivity contrasting target. The ellipsoid has a circular cross section in the electrode plane (with radius 0.1) but with a parameterized semi-axis length, $z=0.1*r$ in the vertical direction. The cylinder had unity conductivity, except for the ellipsoid, with conductivity σ.

A difference signal $s(z,\sigma)=\|\mathbf{v}(z,\sigma)-\mathbf{v}_h\|_2$ was calculated, where $\mathbf{v}(z,\sigma)$ is the vector of EIT measurements using adjacent simulation and measurement for ellipsoid vertical dimension z and conductivity σ, and $\mathbf{v}_h=\mathbf{v}(z,1)$ is the corresponding homogeneous signal.

The signals normalized to their maximum values are shown in Fig˙ 2. A similar effect is shown as before. A flat ellipse gives a much larger signal for conductive contrasts, while, as the ellipse becomes tall, the non-conductive contrast increases (with an equal effect when the ellipse becomes a infinite vertical cylinder).

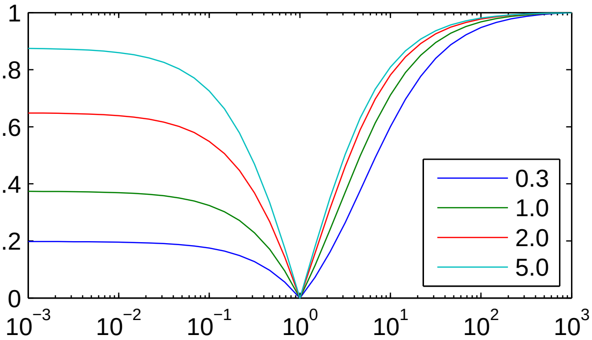

Figure 2: Normalized EIT signal, $s(z,\sigma)$ versus conductivity contrast, σ, for different ellipsoidal vertical semi-axis ratios, r.

4 Discussion

We show that the signal amplitude saturates differently for conductive and non-conductive targets, and this difference depends on the object shape. A complete theory of this effect has been developed by [1], in which the polarization tensors for 2D ellipses (eqn 4.11) and ellipsoids (eqn 4.14) are given.

References

[1] Amari H, Kang H, Polarization and Moment Tensors: with Applications to Inverse Problems and Effective Medium Theory. Applied Mathematical Sciences Series, Vol 162, Springer-Verlag, 2007.