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The Indian Schema as Analogical Reasoning

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Abstract

We investigate the validity of a reading of the Indian Schema as presented within Gotama's Nyāya-Sūtra as Analogical Reasoning within the framework of Pure Inductive Logic.

Key words: Indian Schema, Nyāya-Sūtra, Analogy, Pure Inductive Logic, Rationality.

Introduction

According to J.Ganeri [8, p5] when H.T.Colebrooke first introduced Gotama's so called Hindu Syllogism, subsequently dubbed Indian Schema, in the Nyāya-Sūtra to the West at his address to the Royal Asiatic Society in 1824 (see [7]) it caused a flurry of excitement, not least amongst the main logicians at the time, Charles Babbage, Augustus De Morgan, and particularly George Boole, as attested by his wife Mary Everest Boole, see [4]. For it seemed that here was some independent[1]

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[1] However see [20] for a detailed argument to the contrary.

development of logic within the subcontinent, breaking the Aristotelian monopoly and providing the space for new ideas to develop.

Subsequently however a section of Britain's Victorian Society, perhaps reluctant to acknowledge such advanced thinking in one of its colonies, 'downgraded' the Indian Schema to the status of analogy, an example of reasoning from particular to particular without any genuine soundness or even justification (see for example [2, p380], [8, p11], [15, p87]). To explain this reading more fully we recall (see for example [7]) the (last) three line version of the Indian Schema:

(Aa)	*Where there is smoke there is fire, like in the kitchen.*	
(b)	*There is smoke on the hill.*	$\underline{A}$
(c)	*Therefore there is fire on the hill.*	

In what follows we shall refer to the first part of (Aa),[2] 'Where there is smoke there is fire' as the *Universal* and the second, implicit, 'when there was smoke in the kitchen there was fire' as the *Example*.

If we take the content of (Aa) to be the Example then it could be argued that by the analogy of the hill with the kitchen we should have 'When there is smoke on the hill there is fire', and hence with (b) Modus Ponens gives (c). On the other hand if we take the content of (Aa) to be the Universal, and in so doing make the Example essentially redundant[3] then the argument corresponds to Aristotle's *Socrates is mortal* schema, or more formally a short natural deduction proof as observed by Schayer [27]. Between these extremes there is a reading whereby (Aa) is interpreted as saying that one has experienced of a number of previous smoke filled kitchens, or the like,[4] so with a high probability smoke on that hill implies fire there too. So in this case the argument would seem to be by induction, followed by an application of Modus Ponens.

It is our main purpose in this paper to argue that treating the Indian Schema as a template for analogical reasoning can be justified as rational, whatever was its original intended reading. Our argument is placed in the context of Pure Inductive Logic (PIL), the study of the rational or logical assignment of belief (as probability). Whilst this is, to our knowledge, the first such investigation within this context there exist already several other interpretations of the Indian Schema

[2]Or the corresponding statement in other instances of the schema.

[3]Beyond ensuring that the Universal does apply in some case other the one it is being invoked for, see [18], [19, p16].

[4]So in spirit (Aa) becomes the repeated 'When there *is* smoke in the kitchen there *is* fire' rather than the single experience 'When there *was* smoke in the kitchen there *was* fire.

which diverge from its being simply a fragment of Aristotelian deductive logic, in particular Oetke's interpretation as Nonmonotonic or Default Logic in [23] and Ganeri's as Case Based Reasoning in [9]. In important ways both of these overlap with the picture we shall paint, PIL is naturally nonmonotonic and as we shall see can effectively argue on the evidence of past cases.

A Case for Analogical Reasoning in the Indian Schema

In this paper we argue that treating the Indian Schema as a schema for analogical reasoning can be justified on the grounds of its being in some sense rational; we do not necessarily base this on its original intended reading. Nevertheless we would like to mention some arguments which could point to Gotama originally viewing it as a template for such reasoning.

The main aphorisms in the Nyāya-Sūtra which refer to the 'example' ((Aa) above) are Sūtras 32, 36 and 37. Sūtra 32 just lists the parts of the schema, in its full five line version, and simply calls this part 'example' (*udāharaṇa*). Sūtra 36 aims to expand on this for a homogeneous example (as in (Aa) above) whilst Sūtra 37 acts similarly for a heterogeneous example (to which we will return later). In Sūtra 36[5] translations differ noticeably. Vidyabhusana presents it in [31] as:

> *A homogeneous (or affirmative) example is a familiar instance which is known to possess the property to be established and which implies that this property is invariably contained in the reason given.*

whilst in [13] Jha gives it as

> *That familiar instance, – which through similarity to what is to be proved (i.e. subject), is possessed of a property of that (subject) – constitutes the 'Statement of the Example'*

Certainly the first of these leans towards a reading of the Indian Schema as a pattern of deductive reasoning. However this interpretation is attributed to Dharmakīrti (c600CE-660CE), or possibly his predecessor Dignāga (c480CE-540CE), both of whom wrote several centuries after Gotama's original Nyāya-Sūtra (c200BCE-150CE). In a less suggestive translation, [9], Ganeri gives

> *The example is an illustration which, being similar to that which is to be proved, has its character.*

[5] *sādhya-sādharmyāt tad-dharma-bhāvī dṛṣṭānta udāharaṇam*

Here Vidyabhusana's 'universal' which made the example 'as in the kitchen' redundant has been replaced by 'has its character' or more simply 'is a good example'. With this reading then a form of the Indian Schema more closely reflecting what Gotama actually says might be:

(Ba)	*When there was smoke in the kitchen there was fire.*	
(b)	*There is smoke on the hill.*	$\underline{B}$
(c)	*Therefore there is fire on the hill.*	

together with a rider that (Ba) is a *good or appropriate example in this case.* We shall later return to what we might understand by the phrase in italics.

This interpretation is supported by that of Zilberman in [33, p54] who asserts that the word 'nyāya' initially meant 'setting up as a model' or 'citing example' and Bochenski who, at [3, p430], says that the early formula of the Sūtras was 'simply an argument by analogy'. Both analyse the schema with the Universal absent. Likewise Randle at [26, p.401] claims there was originally a reading (of the five line version) of the schema as analogical reasoning.

Zilberman also gives versions of the schema from the Charaka-saṃhitā, Sustrasthana Adhyay 1, see [33, p53], where there is simply an Example, no hint of a universal.

A further criticism of the Universal reading of (a), as J.S.Mill [21] pointed out in relation to Aristotle's *Socrates is mortal* example, is that it can scarcely be said to generate (true) knowledge which was crucial for Hindu logic (see for example Gupta [10] or Matilal [17, p199]) because to know the Universal would mean one already had to know that smoke on the hill implied fire on the hill.

A final argument in favour of (Ba) rather than (Aa) being Gotama's real intention is that, as aforementioned, it was Dharmakīrti (c600CE-660CE) (or possibly his predecessor Dignāga (c480CE-540CE)) who introduced the Universal reading of the example in the schema.[6] The obvious candidate for its precursor is surely (Ba). This candidate seems all the more reasonable given that Dignāga retained mention of the Example despite its apparent redundancy. An explanation for this, see for example [18, Section 1.7], is that when Dignāga expressed (Aa) in this way he was concerned that the premise of the Universal should not uniquely hold of the subject of the schema, i.e. the hill in the above version. One is tempted to conclude then that Dignāga continued to feel the argument required a different supporting, i.e. analogous, Example. (On this point see also [19, p16–], [30, Sec.3.2].)

[6] In contrast to Nāgārjuna (c250CE-300CE) who in his Upāyakauśalya-Sūtra claims that just the three lines of the thesis, reason and an Example (positive or negative) suffices, see [32, p119].

A Formalization

Putting aside for the present the rider 'is a good example' we wish now to formalize B within the predicate logic framework of Pure Inductive Logic (see [24]). This is not entirely unproblematic. To start with in Nyāya there is no use of symbols. However it seems clear that $\underline{A}$ can be viewed as a template, its use was surely not just to argue about fires on hills and kitchens, so from that point of view 'smoke in the kitchen' can be thought of as a term with parameters 'smoke' and 'kitchen'.

Just what sort of a term however is still problematic. The most natural one from our Western present day viewpoint seems to be the 'smoke', or 'smokiness', is a unary predicate applying to the constant 'kitchen'. In other words with the obvious symbolism 'smoke in the kitchen' becomes $S(k)$.

However other interpretations have been proposed, hardly surprisingly since even if the reading and commentaries on the Nyāya-Sūtra had not themselves evolved the lack of definite and indefinite articles and avoidance of quantifiers in Sanskrit and, according to Müller, [22], the Brahmans' preference for concrete rather than abstract terms already invited enormous freedom of expression. For example that 'kitchen' is the predicate and 'smoke' the constant, or (see [1], [17], [29]) that in 'smoke in the kitchen' the 'smoke' and 'kitchen' are both constant arguments of a binary relation of *pakṣa*, roughly corresponding to 'occurring at'. We plan to investigate these further in a forthcoming paper but for the sake of brevity herein we shall limit ourselves to the first version, $S(k), F(h)$, etc..

The next problem concerns the formalization of the connective in (Ba). Influenced by Dharmakīrti's Universal one's initial thought might be to interpret it as

If there was smoke in the kitchen then there was fire

which with the obvious symbolism renders $\underline{B}$ as

$(\tilde{a})$	$S(k) \to F(k)$	
$(\tilde{b})$	$S(h)$	$\underline{C}$
$(\tilde{c})$	$\therefore F(h)$	

We shall investigate this version though it seems to us possibly flawed in that it would allow as the Example a situation where there was a kitchen with *no* smoke.[7] A possibly more reasonable view of the Example, see for instance [6], [12], [17], [19], [26], is that it refers to a situation in which smoke and fire in the kitchen were perceived as concomitant. With that reading then it seems that the connective in $\underline{B}$ should be a bi-conditional, i.e. 'if and only if', giving the schema

[7] A point we shall return to later when considering an isolated heterogeneous Example.

$$
\begin{array}{lll}
(D\tilde{a}) & S(k) \leftrightarrow F(k) & \\
(\tilde{b}) & S(h) & \underline{D} \\
(\tilde{c}) & \therefore F(h) &
\end{array}
$$

Apart from these two potions one might feel that maybe all the example is really giving us is that smoke and fire happened to occur together in the kitchen, giving the schema

$$
\begin{array}{lll}
(E\tilde{a}) & S(k) \wedge F(k) & \\
(\tilde{b}) & S(h) & \underline{E} \\
(\tilde{c}) & \therefore F(h) &
\end{array}
$$

An immediate criticism of this schema in the present context is that Indian Logicians have been at pains to point out that the concomitance of fire and smoke in the kitchen is not simply by chance, contingent, there is even an implicit causality here. As we shall see later when we consider also the heterogeneous example there are further reasons to reject this formalization.

Our plan is to investigate schemata $\underline{C}$, $\underline{D}$ and $\underline{E}$ in the context of Pure Inductive Logic, more exactly the extent to which they are justified as 'rational'.

Introducing Pure Inductive Logic

The subject of Pure Inductive Logic, PIL for short, (see [24]) is concerned with the rational assignment of belief (as subjective probability) in the absence of any intended interpretation. There is no conceit here that PIL knows what we mean by rational. Rather, in its current state of development it simply formalises within probability logic principles of probability assignment which have some putative claim to being intuitively rational and endeavours to elucidate their mathematical consequences. The intention is not primarily to give philosophical arguments for accepting certain principles; rather, to say that *if one accepts such-and-such principles then by dint of mathematical proof one must accept such-and-such a conclusion.*

Schemata $\underline{C}$, $\underline{D}$ and $\underline{E}$ are not obviously in the form of principles (rational or otherwise) but can be translated to be. To take the example of $\underline{C}$ we can think of it as saying that

> *Given $S(k) \to F(k)$ and $S(h)$ one is at least as justified accepting $F(h)$ as accepting $\neg F(h)$.*

In other words if w is the probability function which represents one's assignment of probabilities in the absence of any knowledge whatsoever then the conditional probability[8] one assigns to $F(h)$ given $(S(k) \to F(k)) \wedge S(h)$ should be at least 1/2, more formally

$$w(F(h) \,|\, (S(k) \to F(k)) \wedge S(h)) \geq 1/2. \tag{1}$$

Similarly schemata $\underline{D}$ and $\underline{E}$ produce the recommendations, or more grandly principles,

$$w(F(h) \,|\, (S(k) \leftrightarrow F(k)) \wedge S(h)) \geq 1/2 \tag{2}$$

$$w(F(h) \,|\, (S(k) \wedge F(k)) \wedge S(h)) \geq 1/2 \tag{3}$$

respectively.

Of course these three 'principles' are given in the context of PIL where w is taken to be one's assignment of beliefs in the absence of any knowledge whatsoever. That is, S, F, k, h (and any other predicates or relations in the language) are entirely uninterpreted. Whilst a mathematicians may be happy to conceive of such a blank slate state a philosopher may argue that in practice we are never in that situation. Indeed any reasonableness of (1), (2), (3) has so far been based on a specific interpretation of S as 'smoke' etc.. With alternative interpretations, for example buying a lottery ticket last Thursday which subsequently won, they could appear far from reasonable.

It is at this point that we would argue that for the purpose of applications this is exactly the content of the rider that (Ba) is a *good or appropriate example in this case.*

Almost invariably when one assigns a probability, for example 1/2 to a coin landing heads, on the basis of some principle, in this case the natural symmetry between heads and tails, one does so ceteris paribus, on the acknowledgement that one's other knowledge is irrelevant to the case in hand. This is the whole point of the rider. It directs that the principle $\underline{B}$, and in turn (1) (similarly (2), (3)) is justified under the assumption that there is nothing special here about 'smoke', 'kitchens' etc. that is relevant. In other words that we can view $\underline{B}$ as (1) in the framework of PIL with S, k etc.. uninterpreted.

Note that this does not move the whole force of the schema onto the rider, making (1), (similarly (2), (3)) essentially worthless on the grounds that the rider already

[8] We shall take it as read that in such expressions as $w(\theta \,|\, \phi)$ the conditional probability is *well defined*, meaning that $w(\phi) > 0$.

requires knowledge of the conclusion (rather like Mill's argument cited above). One certainly could think something was a good example and draw the conclusion prescribed by $\underline{B}$ only to find that because of other factors, unknown at the time, the belief and possibly action based on that belief was ill advised. But that is not the point. PIL is not out to give the *true answer* (whatever that may mean) in the sense that statistics is, nor a maximum utility answer in the sense that decision theory is. PIL is (currently) trying to elucidate what we might mean by a *rational answer* in the circumstances.

Our contention here is that as principles of PIL (1), (2) and (3) are arguably rational and that in consequence, and contra to the Victorian rebuke, even in the form $\underline{B}$ the Indian Schema is justified. Our argument for this purported rationality will be based on (1), (2) and (3) being mathematical consequences of certain other established principles of PIL which have the strongest cases of all for being dubbed 'rational'. Prior to stating these principles though we need to give a fuller explanation of the set-up in PIL.

The Pure Inductive Logic Context

Pure Inductive Logic as described in [24] is conventionally set within a predicate language L with a finite set of relation symbols $P_1, \ldots, P_q$ and countably many constants $a_1, a_2, a_3, \ldots$ and no function symbols nor equality. Let SL denote the set of sentences of L formed as using the usual connectives $\neg, \rightarrow, \wedge, \vee$ and quantifiers $\exists, \forall$. Let $QFSL$ denote the quantifier free sentences in SL.

A probability function on L is a function $w : SL \rightarrow [0, 1]$ such that for $\theta, \phi, \exists x\, \psi(x) \in SL$,

(i) If $\models \theta$ then $w(\theta) = 1$.

(ii) If $\theta \models \neg\phi$ then $w(\theta \vee \phi) = w(\theta) + w(\phi)$.

(iii) $w(\exists x\, \psi(x)) = \lim_{n\to\infty} w\left(\bigvee_{i=1}^{n} \psi(a_i)\right)$.

From these all the expected properties of probability follow (see [24, Prop. 3.1]), in particular if $\theta \models \phi$ then $w(\theta) \leq w(\phi)$.

Given such a w with $w(\phi) > 0$ we can define the conditional probability function

$$w(\theta \mid \phi) = \frac{w(\theta \wedge \phi)}{w(\phi)} .$$

As indicated above, in what follows we will make the a tacit assumption that any *conditional we consider is well defined, in other words that the denominator $w(\phi)$ above is non-zero.*

We wish to elucidate 'rationality constraints' on w in the case when the symbols of L are entirely uninterpreted. In other words if w is to represent a 'rational' assignment of probabilities to the sentences of L what properties in addition to (i)-(iii) should w satisfy.

Numerous such constraints, usually in the form of principles that w should obey, have been proposed based on various intuitions of what 'rational' might mean. The most forceful go back to Johnson [14] and Carnap [5] (or see Carnap's Axioms for Inductive Logic at [28, p.973]) and they are based on symmetry, the idea being that it would be irrational of w to break existing symmetries in the language.

The most obvious symmetry is between the constants, yielding:

The Constant Exchangeability Principle, Ex

If $\theta \in SL$ and the constant symbol a_j does not appear in θ then $w(\theta) = w(\theta')$ where θ' is the result of replacing each occurrence of a_i in θ by a_j.[9]

Similarly using the symmetry between relations of the same arity:

The Predicate Exchangeability Principle, Px

If the relation symbols P_i, P_j of L have the same arity and P_j does not appear in $\theta \in SL$ then $w(\theta) = w(\theta')$ where θ' is the result of replacing each occurrence of P_i in θ by P_j.

Satisfying these two principles are widely considered to be necessary requirements for w to be considered rational. A third symmetry condition is based on the idea that since the context is supposed to be entirely uninterpreted there is symmetry between P_i and $\neg P_i$,[10] just in the same way as there is between heads and tails when we toss a coin. This yields:

The Strong Negation Principle, SN

$w(\theta) = w(\theta')$ where θ' is the result of replacing each occurrence of the relation symbol P_i in θ by $\neg P_i$.

---

[9]This formulation of Ex is equivalent to that given in, say [24], and avoids introducing extra notation.

[10]Since $\neg\neg P_i \equiv P_i$.

The Theorem

We now view $\underline{C}$, $\underline{D}$ and $\underline{E}$ in the context of PIL described in the previous section. Henceforth fix the language L be the language with just two unary relation symbols S, F and the usual constant symbols $a_1, a_2, a_3, \ldots$ of L, two of which we will designate h and k.

Starting with $\underline{C}$ it is clear that $(\tilde{c})$ is not forced by $(C\tilde{a})$ and $(\tilde{b})$ in the sense of being a logical consequence of them. However for the practical purpose of making a decision as to whether or not to accept, or act on $(\tilde{c})$ given $(C\tilde{a})$ and $(\tilde{b})$ it would be enough that $(\tilde{c})$ was assigned a probability of at least 1/2. Read in that way then accepting schema $\underline{C}$ amounts to accepting the PIL principle (1). Similarly accepting $\underline{D}$, $\underline{E}$ amounts to accepting the PIL principles (2), (3), respectively.

The main technical observations in this paper are that:

Theorem 1. *Let w be a probability function on L satisfying Ex+Px+SN. Then*[11]

$$w(F(h) \,|\, (S(k) \to F(k)) \wedge S(h)) \geq 1/2. \tag{4}$$

$$w(F(h) \,|\, (S(k) \leftrightarrow F(k)) \wedge S(h)) \geq 1/2. \tag{5}$$

$$w(F(h) \,|\, (S(k) \wedge F(k)) \wedge S(h)) \geq 1/2. \tag{6}$$

with equality in (4), (6) just if w equals Carnap's c_∞ (i.e. m^*).[12]

The proof of this theorem will be given in the Appendix. One might of course have hoped here for strict inequalities. In fact as we also show in the Appendix we can only have equality in (4) and (6) if w equals Carnap's c_∞ (i.e. m^*), a probability function which denies all inductive influence, giving, for example

$$c_\infty(S(k_{n+1}) \,|\, S(k_1) \wedge S(k_2) \wedge \ldots \wedge S(k_n)) = 1/2$$

no matter how large n is. So to accept Ex+Px+SN as rational but not acknowledge that (4), (6) with strict inequality were also rational would be to, at least partially, advocate c_∞.

Equality in (5) does not force $w = c_\infty$, nevertheless a similar dismissive argument to the one given above can still be made for these additional choices – see the Appendix.

[11] Under the standing assumption that these conditional probabilities are well defined.

[12] The exact conditions for equality in (5) are more involved and are given explicitly in [25]. However equality never holds for the probability functions in Carnap's Continuum except for c_∞.

Theorem 1 then tells us that, subject to the rider[13], it is at least as rational to accept $\underline{C}$, $\underline{D}$ and $\underline{E}$ as it is to accept Ex+Px+SN. To repeat ourselves then, we are not claiming that Gotama and the Naiyyayika scholars might somehow have intuited this theorem, but simply that the theorem in some sense justifies accepting versions $\underline{C}$, $\underline{D}$ and $\underline{E}$ of the Indian Schema.

Theorem 1 immediately suggests a number of other questions. For example does the theorem still hold if we have evidence of many kitchens, not just one? In other words do

$$w(F(h) \mid \bigwedge_{i=1}^{n}(S(k_i) \to F(k_i)) \wedge S(h)) \geq 1/2, \tag{7}$$

$$w(F(h) \mid \bigwedge_{i=1}^{n}(S(k_i) \leftrightarrow F(k_i)) \wedge S(h)) \geq 1/2, \tag{8}$$

$$w(F(h) \mid \bigwedge_{i=1}^{n}(S(k_i) \wedge F(k_i)) \wedge S(h)) \geq 1/2, \tag{9}$$

still follow from Ex+Px+SN? And in similar vein is it the case as one might hope that as m tends to infinity the left hand sides of (7), (8), (9) are monotone increasing and tending to limit 1?

In [25] we give the rather more technical proofs that (7), (8) and (9) also follow from Ex+Px+SN and investigate the general question of when the limit is 1.

The Heterogeneous Example

While the debate about the Indian Schema has concentrated mainly on schema $\underline{A}$, where the example is homogeneous, some accounts (see for example [31, p.287]) consider the case of combining two examples, one homogeneous and one heterogeneous. This is manifest in the schema becoming

(Fa)	*Where there was smoke there was fire, like in the kitchen, unlike on the lake.*	
(b)	*There is smoke on the hill.*	$\underline{F}$
(c)	*Therefore there is fire on the hill.*	

the 'unlike on the lake' citing a heterogeneous example where there is no fire and

[13] Recall that the rider now says that when considering $\underline{C}$, nothing but $(C\tilde{a})$ and $(\tilde{b})$ is relevant, and similarly for $\underline{D}$ and $\underline{E}$.

no smoke.[14,15] This is generally taken to comply with the requirements of Sūtra 37[16] which is presented by Jha, [13, p.429], as:

> *And the other kind of 'Statement of Example' is that which is contrary to what has been described in the foregoing Sūtra,*

by Vidyabhusana, [31, p.12], as:

> *A heterogeneous (or negative) example is a familiar instance which is known to be devoid of the property to be established and which implies that the absence of this property is invariably rejected in the reason given.*

and by Ganeri in [9] as

> *Or else, being opposite to it, is contrary.*

Proceeding with schema $\underline{F}$ as we did with schema $\underline{A}$ previously and using the constant l to denote the lake we arrive at two possibilities corresponding to $\underline{C}$ and $\underline{D}$. Namely:

$$\begin{array}{lll} (G\tilde{a}) & (S(k) \to F(k)) \wedge (\neg F(l) \to \neg S(l)) & \\ (\tilde{b}) & S(h) & \underline{G} \\ (\tilde{c}) & \therefore F(h) & \end{array}$$

$$\begin{array}{lll} (I\tilde{a}) & (S(k) \leftrightarrow F(k)) \wedge (\neg S(l) \leftrightarrow \neg F(l)) & \\ (\tilde{b}) & S(h) & \underline{I} \\ (\tilde{c}) & \therefore F(h) & \end{array}$$

In turn these yield the 'analogy principles'

$$w(F(h) \,|\, (S(k) \to F(k)) \wedge (\neg F(l) \to \neg S(l)) \wedge S(h)) \geq 1/2. \tag{10}$$

$$w(F(h) \,|\, (S(k) \leftrightarrow F(k)) \wedge (\neg S(l) \leftrightarrow \neg F(l)) \wedge S(h)) \geq 1/2. \tag{11}$$

[14]Sometimes 'like on the lake' is used here instead of 'unlike on the lake' to express the same thing!

[15]Purely heterogeneous Examples are sometimes considered, especially when illustrating erroneous, or deserving of rebuke, formulations of the schema but these will essentially be covered herein by what we will conclude vis-a-vis the combined case.

[16]*tad-viparyayād vā viparītam*

Clearly (10) and (11) are equivalent to 'two kitchen' versions of (1), (2) respectively and so both follow from Ex+Px+SN (as mentioned after (7), (8)). Similarly of course the principles given by (10) and (11) but just for the lake are equivalent to (1) and (2) respectively and so also follow from Ex+Px+SN.

The possibility of this purely heterogeneous version however argues against formulation $\underline{E}$ of $\underline{B}$. For capturing the Lake Example by 'no smoke and no fire on the lake' would scarcely encourage one to jump to the conclusion there was fire on the hill because there was smoke there. And indeed the corresponding 'principle'

$$w(F(h) \mid \neg S(l) \wedge \neg F(l) \wedge S(h)) \geq 1/2$$

is actually without any power since under the assumption that w satisfies Ex+Px+SN the left hand side (when defined) is always exactly 1/2.

In summary then we would claim that each of the PIL formalizations (1),(2), (10),(11) of the Indian Schema can be justified as rational on the grounds that they follow from the widely accepted symmetry principles Ex+Px+SN.

Conclusion

In this paper we have argued that, whether or not it was Gotama's original intention, one can view the Indian Schema as a principle of analogical reasoning and furthermore in the context of Pure Inductive Logic to adopt it is as rational as respecting simple symmetries when assigning (subjective) probabilities. Consequently it could be said that those Victorians who viewed the Indian Schema as no more than arguing by analogy from particular to particular were dismissing it too readily. Analogical argument may not have the force of deductive argument but nevertheless can claim to be both rational and widely applicable.

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Appendix

Proof of Theorem 1

Let $\alpha_1(x) = S(x) \wedge F(x)$, $\alpha_2(x) = S(x) \wedge \neg F(x)$, $\alpha_3(x) = \neg S(x) \wedge F(x)$, $\alpha_4(x) = \neg S(x) \wedge \neg F(x)$. Then writing $\alpha_1\alpha_2$ etc. for $w(\alpha_1(h) \wedge \alpha_2(k))$ etc., (1) becomes

$$\frac{\alpha_1^2 + \alpha_1\alpha_3 + \alpha_1\alpha_4}{\alpha_1^2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 + \alpha_2\alpha_1 + \alpha_2\alpha_3 + \alpha_2\alpha_4} \geq \frac{1}{2}, \tag{12}$$

(2) becomes

$$\frac{\alpha_1^2 + \alpha_1\alpha_4}{\alpha_1^2 + \alpha_1\alpha_4 + \alpha_2\alpha_1 + \alpha_2\alpha_4} \geq \frac{1}{2}\,. \tag{13}$$

and (3) becomes

$$\frac{\alpha_1^2}{\alpha_1^2 + \alpha_1\alpha_2} \geq \frac{1}{2}. \tag{14}$$

Under the assumption Ex we have $\alpha_i\alpha_j = \alpha_j\alpha_i$ for any distinct $i, j \in \{1, 2, 3, 4\}$. Under the assumption Ex + Px moreover

$$\alpha_2^2 = \alpha_3^2, \quad \alpha_1\alpha_2 = \alpha_1\alpha_3, \quad \alpha_2\alpha_4 = \alpha_3\alpha_4,$$

and under the assumption Ex+Px+SN also

$$\alpha_1^2 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2, \quad \alpha_1\alpha_4 = \alpha_2\alpha_3, \quad \alpha_1\alpha_2 = \alpha_1\alpha_3 = \alpha_2\alpha_4 = \alpha_3\alpha_4.$$

For (12) to hold for a probability function satisfying Ex+Px+SN we thus require that

$$\frac{\alpha_1^2 + \alpha_1\alpha_2 + \alpha_1\alpha_4}{\alpha_1^2 + 2\alpha_1\alpha_4 + 3\alpha_1\alpha_2} \geq \frac{1}{2},$$

equivalently, $\alpha_1^2 \geq \alpha_1\alpha_2$.

By [11, Lemma 6], or an exactly analogous argument to that given at [24, p89] for Ax, any function w satisfying Ex+Px+SN can be expressed as an integral using the probability functions $w_{\langle x_1,x_2,x_3,x_4\rangle}$, where the x_i are nonnegative real numbers satisfying $x_1+x_2+x_3+x_4=1$, and

$$w_{\langle x_1,x_2,x_3,x_4\rangle}(\alpha_{i_1}(a_{j_1})\wedge\alpha_{i_2}(a_{j_2})\wedge\ldots\wedge\alpha_{i_m}(a_{j_m}))=x_{i_1}x_{i_2}\cdots x_{i_m}$$

as follows:

$$\begin{aligned} w &= 8^{-1}\int w_{\langle x_1,x_2,x_3,x_4\rangle}+w_{\langle x_1,x_3,x_2,x_4\rangle}+w_{\langle x_4,x_2,x_3,x_1\rangle}+w_{\langle x_4,x_3,x_2,x_1\rangle} \\ &\quad +w_{\langle x_2,x_1,x_4,x_3\rangle}+w_{\langle x_2,x_4,x_1,x_3\rangle}+w_{\langle x_3,x_1,x_4,x_2\rangle}+w_{\langle x_3,x_4,x_1,x_2\rangle}\,d\mu(\vec{x})\end{aligned}$$

where μ is some normalized σ-additive measure on the set of $\vec{x}=\langle x_1,x_2,x_3,x_4\rangle$ as above.

Hence it suffices to show that for all $\langle x_1,x_2,x_3,x_4\rangle$,

$$2x_1^2+2x_2^2+2x_3^2+2x_4^2\geq 2x_1x_2+2x_1x_3+2x_2x_4+2x_3x_4, \tag{15}$$

which clearly holds (with equality just if $x_1=x_2, x_1=x_3, x_4=x_2, x_4=x_3$). Equality in (12) can occur only when equality in (15) holds for a μ-measure 1 set of $\vec{x}$, i.e. when $w=c_\infty$.

Turning to (13), this is equivalent to

$$\alpha_1^2+\alpha_1\alpha_4\geq\alpha_1\alpha_2+\alpha_2\alpha_4=2\alpha_1\alpha_2.$$

Using the same trick as for (12) it is enough to show that for such $\langle x_1,x_2,x_3,x_4\rangle$,

$$2x_1^2+2x_2^2+2x_3^2+2x_4^2+4x_1x_4+4x_2x_3\geq 4x_1x_2+4x_1x_3+4x_2x_4+4x_3x_4.$$

But this amounts to

$$(x_1+x_4-x_2-x_3)^2\geq 0, \tag{16}$$

so it holds (with equality just if $(x_1+x_4=x_2+x_3)$.

In this case probability functions other than $w=c_\infty$ will give equality rather than a strict inequality. However it is straightforward to see that in that case we must have $x_1+x_4=x_2+x_3=1/2$ for a μ-measure 1 set of $\vec{x}$.

It follows that

$$\int(x_1+x_4)^r(x_2+x_3)^{n-r}\,d\mu(\vec{x})$$

must take the same value, necessarily 2^{-n}, for any $0\leq r\leq n$. In turn this gives that

$$w(F(k_{n+1})\leftrightarrow S(k_{n+1})\,|\,\bigwedge_{i=1}^{n}(F(k_i)\leftrightarrow S(k_i)))=1/2$$

for $n \geq 0$. Informally then, no matter how much evidence there was that in a kitchen fire and smoke were invariably linked w would not alter the value it gave to fire and smoke being linked in the next kitchen encountered.

Finally (14) also reduces to $\alpha_1^2 \geq \alpha_1\alpha_2$. Hence it holds, and with strictly greater than unless $w = c_\infty$.