The Resource Valuation and Optimisation Model – Real Impact from Real Options

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ABSTRACT

This paper presents the scientific framework underpinning the resource valuation and optimisation model (RVOM). The RVOM is a partial differential equation based real options software package, which helps mine owners optimally plan their operations, understand their project risks, and make defensible valuations. This is achieved in the presence of both financial and physical uncertainty, as well as processing capacity constraints. The RVOM can be applied to any multi-ore mine, open pit or otherwise, where the block-order of extraction has already been planned using existing software tools such as the Gemcom Whittle™ strategic mine planning package. The RVOM can also take account of multiple commodities within a single mine, and multiple forms of price behaviour. The three key outputs from the RVOM are: valuation, optimal decision and probability of decision. A decision takes account of upfront costs, and can include an unlimited number of transitions between: normal operation, expanded operation, care and maintenance and abandonment (and variants thereof). The RVOM then employs stochastic process theory to determine the probability of reaching these decisions. This gives the mine operators clear indications as to where their risks lie, aiding their mine planning. A clear example of the RVOMs usage to a case-study gold mine is presented, demonstrating its broad applicability, the added value it can create and how users can easily make use of the RVOM's state-of-the-art algorithmic engine.

INTRODUCTION

Prior to extraction, a mining company must produce a feasible extraction plan to ascertain just how cost-effective the operation is expected to be, if at all. The question then arises, is it possible to vary this plan to make more profit, and if so, is it possible to optimise this gain? To approach a truly optimal extraction plan, one cannot simply examine the physical production schedule, or the underlying economics, in isolation. Instead both of these facets must be simultaneously considered. This is because an optimal extraction plan will require the inherent financial constraints and uncertainties to feedback into the production schedule, which in turn affects the resulting cash-flows and profit. The intertwined nature of this problem gives rise to complexities not common within most studies across the broad fields of finance or process optimisation, which is perhaps a contributing reason to why optimal resource extraction has been such a challenging problem to solve. However, by utilising tools from areas of mathematics, computer science, probability theory, economics and mine engineering constraints, this paper presents a model which attempts to sit in the middle of both of these aspects, to help solve this optimisation problem. This model and software package is named the resource valuation and optimisation model (RVOM).

There have been many important previous contributions to solving the problems of either maximising the mine valuation by optimally managing the extraction plan under economic

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uncertainty, or minimising the impact of geological uncertainty in mine scheduling, and sometimes both (Newman *et al,* 2010 give a comprehensive review of such studies). Many of these studies emanate from the economics research area of real options, which is the study of business decisions under uncertainty – see Dixit and Pindyck (1994) for a broad overview of the subject. In fact, one of the seminal papers in real options was by Brennan and Schwartz (1985), on the optimal management of a natural resource extraction project. Their paper showed how to calculate the optimal criteria for when to abandon or mothball an extraction process, given an uncertainty in the future price and capital costs for switching, which would maximise the mine's valuation. To achieve this they used partial differential equations, but assumed a fixed ore grade and production schedule, and neglected ore-processing considerations. Numerous studies have followed on from their work, but all have essentially remained true to Brennan and Schwartz's fundamental economics, which perfectly captures the principles of real options theory.

Parallel to this classic real options approach, other methods to help solve the mine scheduling problem (which could easily take account of grade variation and the finiteness of resource) were developed. The highly acclaimed Lerchs-Grossman algorithm (Lerchs and Grossman, 1965) used graph theory to give an excellent method for finding the optimal ultimate pit-shell shape of an open pit mine, when no discounting or uncertainty was present. The algorithm was then extended by Whittle (1988) and now exists as part of the Gemcom Whittle™ software package (Gemcom, 2011). Gemcom Whittle™ gives mine planners insight into the ordering of pushbacks until the ultimate pit-shell shape is reached (Gholamnejad and Osanloo, 2007), and is widely used across the global mining industry (Whittle and Cahill, 2001). The Lerchs-Grossman algorithm was extended by Khalokakaie, Dowd and Fowell (2000) to include variable slope constraints in the ultimate pit shape, which allowed different geologies within the same mine to be taken into account. The problem of how to include discounting within the ordering of block extraction was considered by Tolwinski and Underwood (1996) by using dynamic programming, in two dimensions. However, as Tolwinski and Underwood explained, when discounting is included, the solution of the Lerchs-Grossman algorithm is no longer always optimal. This is because the time value of money affects the very nature of the problem, and instead Tolwinski and Underwood proposed an approximate dynamic programming method to tackle this increased complexity. The difficulty of finding the optimal pit shell shape (or sequence of extraction) in the presence of discounting means that including financial uncertainty can only compound matters further; in particular, if the operator has the option to abandon the operation, then the presence of economic uncertainty implies one does not know with certainty the final pit shell shape (for an empirical study of this, see Moel and Tufano, 2002).

The inclusion of geological uncertainty into mine scheduling has been studied by (amongst many others) Dimitrakopoulos, Martinez and Ramazan (2007), Menabde *et al* (2004) and Ramazan and Dimitrakopoulos (2004). These specific examples all analyse equally probable orebody simulations, where a constraint is to restrict the probability of a downside event, such as minimising the possibility of a sustained period of extracting ore much below the expected quality. Once a tolerable schedule has been produced, one can investigate how the schedule impacts upon valuation. Ideally, such an impact needs to be paired with economic considerations as well, as highlighted by Martinez (2009) and Monkhouse and Yeates (2005). Whilst these more recent studies capture a fuller picture of mining operations, they have generally relied upon mixed integer programming, Monte Carlo techniques or genetic algorithms (for example, Myburgh and Deb, 2010) to derive solutions. These techniques can carry with them a sizeable time penalty in extracting solutions (Caccetta and Hill, 2003; Dimitrakopoulos, Farrelly and Godoy, 2002) and the time penalty can be magnified further when one tries to optimise each decision available to an operator.

In all of the above studies, there are both advancements made, and limitations produced. However, it is the aim of this paper to select the appropriate advancements that can be simultaneously used within a classic real options framework, and to show how they are incorporated within the RVOM. Despite much hype around the utility of real options, the methodology is not a silver bullet to all problems concerning uncertainty. However, when the problem permits – and the mathematical tools to extract solutions have been developed – real options can be used to provide insight into optimal management under uncertainty and realisable added value. In this same manner, the RVOM has been created to give an accurate real options analysis for a broad class of mining operations, and uses advanced mathematics to extract useful information from operational data, without heavy computational time penalties being incurred.

The core requirement behind the RVOM is that a block-by-block processing schedule has been produced, using a tool such as Gemcom Whittle™. This means that the RVOM is not, in effect, proposing a radical change in scheduling practice. Rather, it is a tool to enable detailed, accurate and operationally complete comparisons for any given schedule – this is why the RVOM is a natural complement to a mine planner who uses Gemcom Whittle™. If more than one schedule is produced, then the RVOM is able to accurately compare this set of feasible pathways, whilst ensuring a high level of operational constraints are captured within the modelling. This accuracy is a reflection of the RVOM's ability to take account of multiple operational decisions (abandon, enter care and maintenance period and expand, etc), multiple commodity price behaviours (such as geometric Brownian motion and mean-reversion), and processing capacity constraints. With all of these available options, uncertainties, schedules, and standard operational inputs (such as extraction costs), the RVOM gives three core outputs. These are:

- project valuation,
- optimal price for when to take each decision, and
- probability of ever having to take a specific decision.

These outputs give the operator a deep insight into the economic viability, optimal strategy for exercising their available decisions, and quantify the risks associated with the uncertainties present. The interdisciplinary work underpinning the RVOM is detailed in Evatt *et al* (2011), Moriarty *et al* (2011), Johnson *et al* (2011) and Evatt *et al* (2010). Elements of each paper are discussed in subsequent sections of this current paper.

WHAT TYPES OF MINE IS THE RESOURCE VALUATION AND OPTIMISATION MODEL COMPATIBLE WITH?

The RVOM is primarily designed to work with open pit mines, utilising the output from the Gemcom Whittle™ software package. As such, the RVOM is designed to take account of many of the same inputs, including processing capacity constraints, differing processes and extraction costs for different blocks and multiple commodities in each mine. Whilst there is no limit to the number of commodities the RVOM can handle, it can realistically only allow up to two commodities to have an uncertain future price for each computational run. For mines with more than two commodities, the RVOM treats the price of the remaining commodities to be fixed (in the same manner as Gemcom Whittle™ does), and thus does not discard their presence. The user is able to select which two future commodity prices are uncertain, and then sequentially examine the effect of each commodity upon the operation, thus determining the most influential ones.

The fact that the RVOM relies upon a processing schedule, means it could also be tailored to be made applicable to underground mining operations, which also face economic uncertainties and operational decisions.

WHAT UNCERTAINTIES DOES THE RESOURCE VALUATION AND OPTIMISATION MODEL CONSIDER?

The RVOM is designed to be adaptable to various situations and user requirements. It does this to be applicable to multiple mining situations, where different minerals will have different associated economic uncertainties and mine-specific physical uncertainties. To make the RVOM as general as computationally possible, it offers the user a selection of economic uncertainties to choose from.

Price uncertainty

The economic uncertainties available from the RVOM are all forms of 'Ito diffusions' (such as Brownian motion). This is a consequence of their popular usage, their vast applicability to many financial situations (Wilmott, Howison and Dewynne, 1995), and the fact that they can serve as decent proxies to more complex stochastic processes. An Ito diffusion is composed of two parts: the drift of the process and the random noise added to this drift. The most common form of Ito diffusion in finance is geometric Brownian motion (GBM), which can be written as:

$$dS = \mu S dt + \sigma S dB \quad (1)$$

where:

S is the sale price

μ is the drift

σ is the volatility

dB is a small increment of a Wiener process (this represents uncertainty)

dt is a small increment in time

Whilst this particular process is highly utilised to a successful degree, it will not always be a suitable process to use for every commodity, as some commodities follow mean-reverting processes (McCarthy and Monkhouse, 2002). Consequently, in addition to GBM, the RVOM also offers the user the option to select the price to follow a mean-reverting process, in the form:

$$dS = \kappa\,(\alpha - S)\,dt + \sigma S dB \tag{2}$$

where:

κ is the speed of reversion

α is the long-term average of the price

In addition to these two processes, the user can also select the price to follow arithmetic Brownian motion, or a Cox, Ingersoll and Ross (CIR) process, which are respectively:

$$\begin{aligned} dS &= \kappa\,(\alpha - S)\,dt + \sigma\sqrt{S}\,dB \\ dS &= \mu dt + \sigma dB \end{aligned} \tag{3}$$

Because the RVOM allows the user to select up to two uncertain ore prices, the user can choose any two of these four processes for each run of the code, where the prices are allowed to be correlated.

Physical uncertainty

The physical uncertainty caused by the estimated ore grade and processing order variations, is not as clear-cut a quantity to capture as it is for the financial uncertainty. In fact, its quantification is quite subjective. There are a variety of reasons for this, including the fact that the uncertainty can come from multiple sources, such as data gathering, laboratory testing, the kriging process and unplanned variations in the order of extraction. Another reason is that, to a certain degree, the mining company involved can themselves control the uncertainty; a larger investment in improved ore grade measurement would (generally) imply a reduced uncertainty in the estimated ore grade (see Farmer, Fowkes and Gould, 2010 for an example of this in the oil industry). However, it is possible to attempt an aggregation some of these individual uncertainties and then use stochastic simulation to recreate feasible 3D orebodies (Journel, 1974). The objective of including an uncertain ore grade is not merely to find a more representative valuation, as it is often used to minimise the risk of deviation from the production schedule caused by unforeseen subquality ores (see Leite and Dimitrakopoulos, 2007 for an overview). However, once a mine schedule satisfying a specified risk tolerance has been produced, it is then possible to capture the implied impact on option valuation left by the residual uncertainty; this is where the RVOM comes in.

When given a feasible schedule, Evatt *et al* (2010) showed that the problem of capturing the implied economic effect of grade uncertainty upon an option value can be reduced from three spatial dimensions to one. This is because it is not until sale (or processing) that the actual uncertainty is financially realised, and therefore the real options aspect of the problem can be strictly viewed in 1D. By looking at the problem in this way, the RVOM analyses the estimated ore grade of the blocks chronologically, as opposed to spatially; this is highlighted in Figure 1 for a particular example gold mine. If we decide to treat the data as uncertain, then (as this figure suggests) the estimated ore grade can be interpreted to follow a form of mean-reverting stochastic process; the estimated data is just one possible realisable scenario. In the same manner as that proposed by Evatt *et al* (2010), the RVOM models this uncertainty as a CIR process, although other mean-reverting processes could equally be trialled. This then allows for the data-specific physical uncertainty to be quantified from the data itself (as recommended in Smith, 2001), and we choose to do this via maximum likelihood estimates (MLE). These MLEs allow the uncertainty to be parameterised, and thus equally-probable ore processing schedules to be produced. The selection of a CIR process provides a robust first approximation of the physical uncertainty; it provides identifiably similar equally probable ore grades

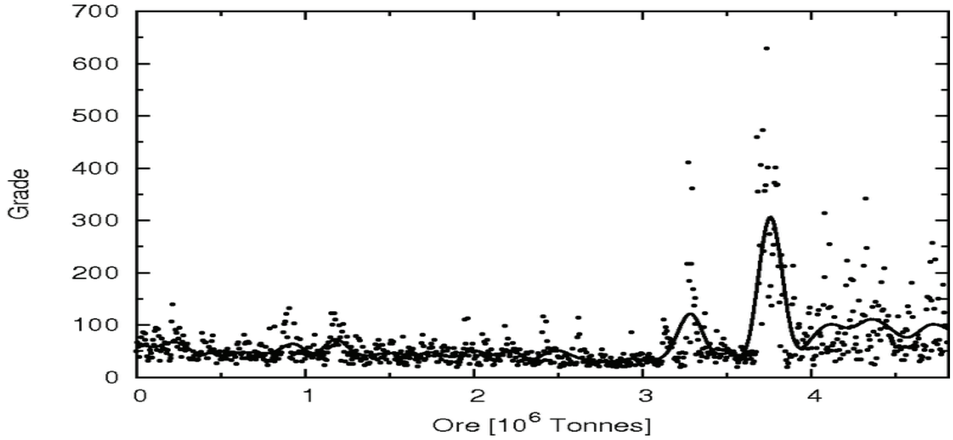

FIG 1 - Real ore grade data from a gold mine from individual blocks is shown as dots. Smoothed average ore grade is shown as solid line.

which are strictly positive, and allows for dynamic variations in the ore grade along the chronological processing path to be captured (as well as providing tractable solutions).

Using the ore grade data of Figure 1, an equally probably simulation is shown in Figure 2. As Figure 2 shows, the simulation bears a defensible resemblance to the original data; it qualitatively captures possible future variations in both ore quality and the timing of processing. An implication of this close similarity (as opposed to different possible future price simulations), is that one must expect the residual physical uncertainty to have a relatively small effect upon an option valuation. This phenomenon is found across many areas of finance, where extra stochastic variables (on top of price uncertainty) have diminishing effects upon valuations. This stems from the fact that the price can follow a much more highly variable path (even if the price is mean-reverting) than the physical uncertainty can (for typical parameter values), and thus it is the price uncertainty that picks up the majority of added value behind each option.

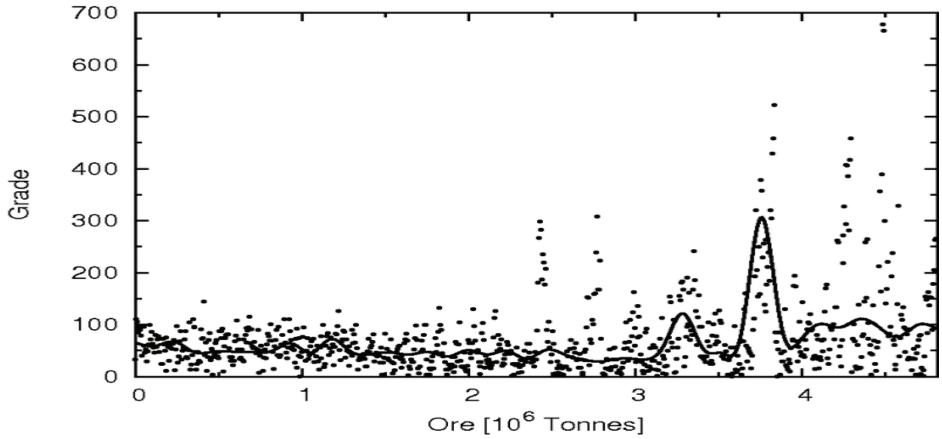

FIG 2 - A set of simulated ore grades from a single run using Monte Carlo methods. Smoothed average ore grade shown as a solid line.

OPERATIONAL STATES

The RVOM is designed to be compatible with a broad array of operational states, for which the mine operator is free to decide when to enter. Operational states may include normal (initial) processing capacity, or expanded processing capacity, or a care and maintenance period, or abandonment. An unlimited number of variations/combinations of each of these states can also be included within the RVOM. Each possible transition between states must be specified to capture whether or not transitions can exist or not, complete with the associated cost of switching (eg it may be possible to move from state A to B, but not back again from B to A). To achieve this generality, the RVOM requires the user to specify each state and possible transition in a vector format, such as:

State: [State Number; State Parameter 1; State Parameter 2; State Parameter 3...]

Transition: [Starting State Number; New State Number; Switching Cost]

Possible state parameter inputs can include: processing capacities, rock types, processing costs, extraction costs and times when such states can, or cannot, be entered into.

For a simple example, let us suppose the mine can only exist in two possible states: normal operation and abandoned. The only available decision is therefore to abandon, so only one possible transition needs to be defined. If we denote normal operation as state '0' and abandoned as state '1', and specify a processing capacity of 10 Mt/a, and the cost for abandoning the operation is US$30 M, then the input vectors would be of the form:

Normal State: [0; 10 000 000; 3]

Abandoned State: [1; 0; 0]

Transition from Normal to Abandon: [0; 1; 30 000 000]

Suppose we can now also include the option to expand, where the new processing capacity and cost double from that of the normal state. However, once expanded the operator cannot return to normal operation, but they can abandon from it. If we define the expanded state as state '2', and the cost for expanding is US$40 M, and the cost for abandoning from the expanded state is US$35 M, then the extra vectors we would now need to include are:

Expanded State: [2; 20 000 000; 6]

Transition from Normal to Expanded: [0; 2; 40 000 000]

Transition from Expanded to Abandoned: [2; 1; 35 000 000]

Because of the ease of this state and transition specification, there is no upper limit to the number of possible operational states, provided they can all be written in such a vector format.

One option that the current version of the RVOM intentionally takes as fixed from the outset, is the decision of whether or not to process each individual block. This deterministic decision making has been selected so as to ensure that the RVOM is compatible with the reality of physical ore extraction. If this decision making was a function of the (uncertain) future price, then it would not allow the day-to-day mine operations to follow a clear extraction plan, and (most importantly) would not allow the mine to maintain a constant processing capacity – unless the mine was able to continuously vary the rate of extraction arising from each and every fluctuation in the market price. That said, in the pure world of mathematics, the RVOM could be altered to include the optimisation of this decision (Johnson *et al,* 2011), but for the mentioned reasons it is not included within the current version of the RVOM.

RESOURCE VALUATION AND OPTIMISATION MODEL OUTPUTS

The RVOM has three core outputs, designed to give a mine operator a clear insight into how they should optimally take their mining decisions, and the associated risks behind such decisions. These outputs are discussed below.

Optimal valuation

A real options based valuation gives the contract owner an optimal expected Net Present Value (NPV); this may be thought of as the optimal expected future cash-flows. To find this optimal NPV, the RVOM needs to calculate the optimal prices for when to take all the available decisions.

Optimal decision criteria

The RVOM is designed to quantify the optimal criteria for when to take the user-specified decisions. By taking these decisions at the optimal points in the extraction process, they allow the mine operator to increase the likelihood of reaping higher cash flows over the long run. This highlights that the RVOM will be of use even to mine owners who are not required by law to produce defensible annual valuations, as they will still be interested in increasing their expected cash flows.

Probability of decision

The probability solver of the RVOM is the most novel and advanced aspect of the underpinning mathematics. By calculating the probability of taking a particular decision, it allows the mine owner insight into the planning risks behind each option. For example, suppose a mine operator wishes to explore the potential impact of including the option to expand their operation. Whilst calculating the value of this option allows a mine owner to weigh up the cost-benefit, the probability of having to take this decision allows the mine planner insight into the level of readiness needed to smoothly take this decision; a 99 per cent chance of expansion implies the mine operator should make firm preparations for a new processing plant; a one per cent chance of expansion means the mine operator can prioritise other planning considerations. In addition to the utility of understanding event probabilities for the mine operator, it will also be of use to regulators, or contract sellers, who are required to undertake a risk-assessment of the economic/environmental/sociological impact of proposed mining ventures.

Work upon the probability of taking a decision under uncertainty – with application to the mining industry – can be found in Evatt *et al* (2011) and Moriarty *et al* (2011). What is interesting about the latter paper, is that it shows that the probability of exercise can have qualitatively different behaviour to the valuation, implying one cannot capture the probability of taking a decision without explicitly calculating it.

THE UNDERPINNING MATHEMATICS

In the same manner as Brennan and Schwartz (1985), this paper uses partial differential equations (PDEs) to frame this optimal extraction problem. This is because PDEs are extremely adaptable to many quantitative situations, including those under uncertainty. They allow one to deeply understand why it is a system behaves as it does (something which a Monte Carlo approach does not), and crucially are perfect for optimising dynamical systems. PDEs do come with a price however, as they can be relatively difficult to use (to the non-expert) and they often rely upon advanced computational techniques to extract solutions. But once they have been solved they are exceptionally powerful and ideal for use in all manner of real options problems.

As already shown, the types of uncertainty which the RVOM is compatible with are Ito diffusions. This quite general type of uncertainty allows us to exploit the Feynman-Kac framework, which shows how a quantity which is a function of an Ito diffusion can be written as the solution to a PDE. A full derivation of this approach is given in Evatt *et al* (2011), in which is shown how the equations describing both valuation and event probability can be derived and solved; it is that which makes this approach so appealing to use. In Evatt *et al* (2011) it is explained how probability of abandonment is an easily interpreted measure of risk in its own right, which aids users in the planning (and justification) of operations. The understanding of event probability was extended further by Moriarty *et al* (2011), in which it was shown how the probability of mine abandonment could have qualitatively different behaviour from that of valuation alone; one cannot infer the probability of making a certain optimal decision by only studying how the valuation is dependent upon the model inputs.

From the underlying probability theory and PDEs, the RVOM extracts solutions using advanced numerical methods. Such a method incorporated into the RVOM methodology is the semi-Lagrangian technique, as used successfully by Chen and Forsyth (2007 and 2010) in relation to optimal natural gas storage operations. The technique allows smooth and stable solutions to be produced when a storage element is present within a valuation, and without its use a 'standard' finite-difference methodology to solving such storage related problems will often break down.

By combining the underlying probability theory with PDEs, and solving them using advanced numerical schemes, the RVOM is able to deliver stable results in fast times. Typically for a one commodity mine with four operational states it would take the RVOM under ten seconds to evaluate, and this time is independent of the number of blocks within the mine. The independence from the number of blocks is a consequence of two factors. Firstly the block extraction pathway has already been determined, and secondly the RVOM is then able to use a suitable data-driven moving average of ore grade, in which blocks are locally aggregated whilst making sure the effect upon valuation is only a fraction of one per cent. This speed of solution allows planners to vary and test multiple different extraction possibilities, and investigate where their relative risks and benefits lie.

EXAMPLE VALUATION

We now apply the RVOM to a example gold mine, whose data has been supplied by Gemcom Software International. This data set is known as the 'Marvin' data set, whose ore grade (along the path of extraction) is shown in Figure 1. The processing rate is 10 Mt/a and at this rate the mine operation has a maximum lifetime of 4.8 years. The mining cost are US$1.5 per tonne, and the processing costs depends upon the three type of rocks present (denoted by the acronyms FR, TR and OX) and they are US$22, US$18 and US$17 respectively, and their recovery ratios are 0.9, 0.92 and 0.94. The discount rate to be used is eight per cent per year, and the the reference gold price is US$25.72 per gram. This gold price is rescaled to a reference price of one. There is an upfront capital cost of US$50 M.

The parameters that are required to capture the value added by taking account of uncertainty and the associated real options available to the mine operator need to be prescribed. The gold price is assumed to follow a mean reverting CIR process, with a mean price level of 95 per cent of the scaled reference price, speed of reversion 0.01, and volatility of 40 per cent. Next the mine is supposed to have three states in which the mine operator must decide where to exist: normal operational, expanded operation, abandonment. The available transitions and costs are: US$20 M to abandon from normal, US$20 M to expand from normal operation, and US$25 M to abandon from expanded operation. The decision for each transition is allowed to be taken at any time or stage along the production schedule.

The first output produced is the valuation, shown in Figure 3. To highlight how a valuation depends upon the volatility in price, the valuation for four different price volatilities: zero, ten, 20, 30 and 40 per cent is shown. The valuation increases monotonically with volatility. The extra value captured by considering the uncertainty and optimal decisions compared to one where the underlying price is fixed (ie the difference in value between a 40 per cent price volatility and a zero per cent price volatility), is around 12 per cent at the reference price, and increases further for lower prices (this difference would obviously increase further if the price volatility was higher, and would also increase if the price process followed a geometric Brownian motion).

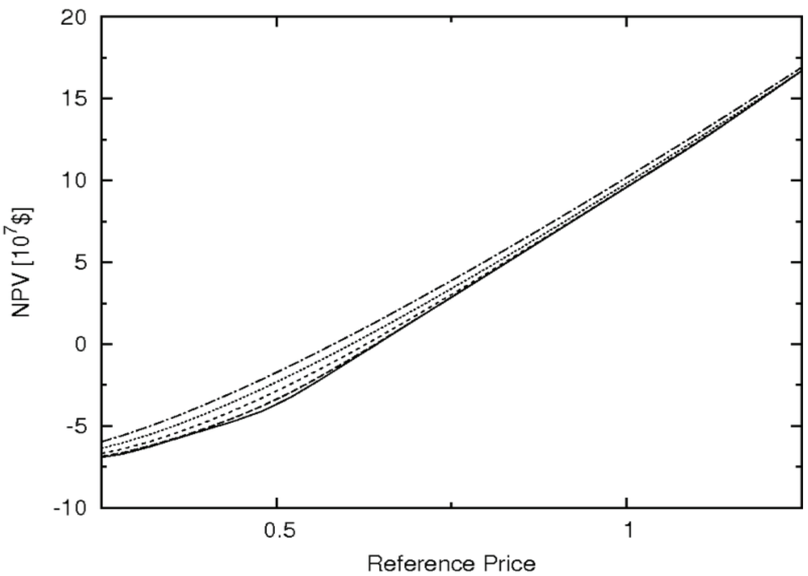

FIG 3 - The net present value for the gold mine with different values of volatility in the underlying commodity price. The figure shows, from bottom to top, volatilities of 0, 10, 20, 30 and 40 per cent.

The next output is the optimal decision-making criteria. This is highlighted in Figure 4, which shows the optimal price for when to expand, abandon from expanded, and abandon from normal operation. More strictly, these optimal prices form a surface in ore-time space, but to aid visually, a cross-section of this surface for a suitable characteristic is shown. The variations in the optimal price, are consequences of the variations in ore grade, and thus one can clearly see how the different stages of mining impact upon when a particular decision is optimal to be taken.

The final core output is the probability of abandonment (the user could just as easily have selected to view the probability of expansion). This probability is shown in Figure 5, and the two possible values relate to whether the mine is currently operating at an expanded rate (dashed line) or at normal operation (solid line). It is clear that by the operator including the probability of expansion, when the price is at the reference level, the risk of ever having to abandon the mine is significantly

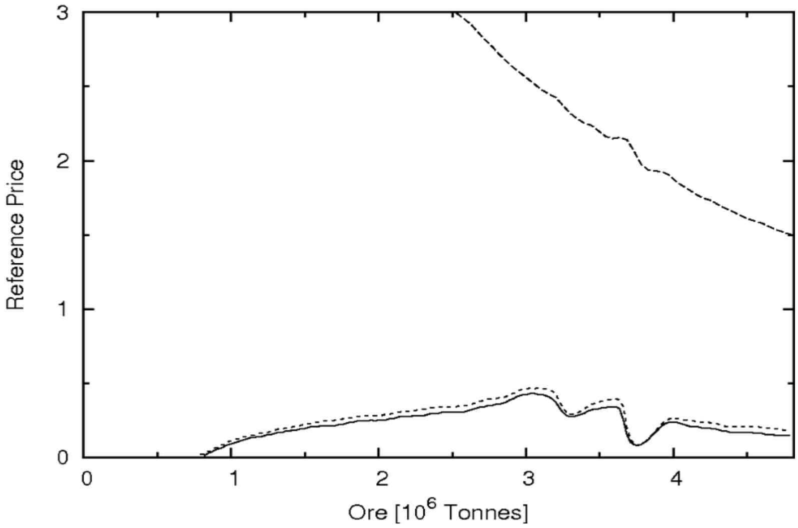

FIG 4 - The optimal decision criteria for the gold mine. The solid line indicates the reference price at which one would abandon the operation if one is in the normal operating mode, whereas the dotted line indicates the price at which one would abandon if operating in the expanded state. The dashed line shows when to expand operations.

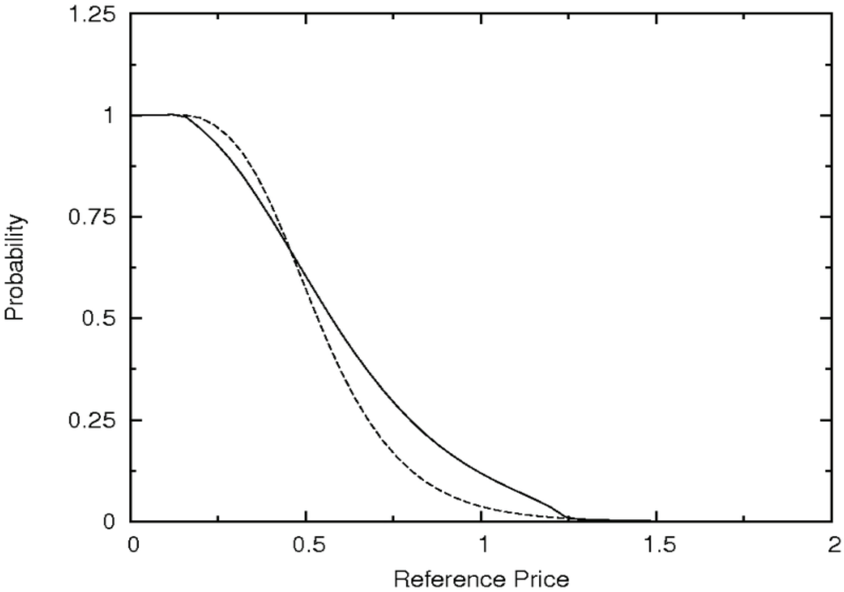

FIG 5 - Shown here is the probability of abandoning operation during the life of the mine given a starting reference price. The solid line shows the probability if one begins in the normal operating mode, whilst the dashed line shows the probability if starting in the expanded state.

diminished; in this case from around 14 per cent to four per cent. It is of interest that the influence of current operating state reverses the relative impact of probability of abandonment for prices below 0.5, and it is effects such as this which makes having a tool which can accurately quantify the complex nature of decision probabilities so appealing to use.

The final graph, Figure 6, shows the impact of including the RVOMs interpretation of physical uncertainty into a valuation. The graph shows the percentage difference between the value of a mine with no physical uncertainty and one where physical uncertainty is included. As expected, the relative size of this economic difference is small compared to the overall valuation; at the reference price level, the inclusion of ore grade uncertainty increases the valuation by approximately one per cent. It is of note that the percentage impact locally increases for regions of prices where one would be more likely to optimally take a decision.

DISCUSSION

This paper has presented the resource and valuation optimisation model (RVOM). The RVOM is a real options tool aimed at helping mine operators produce extra revenues and quantify their project

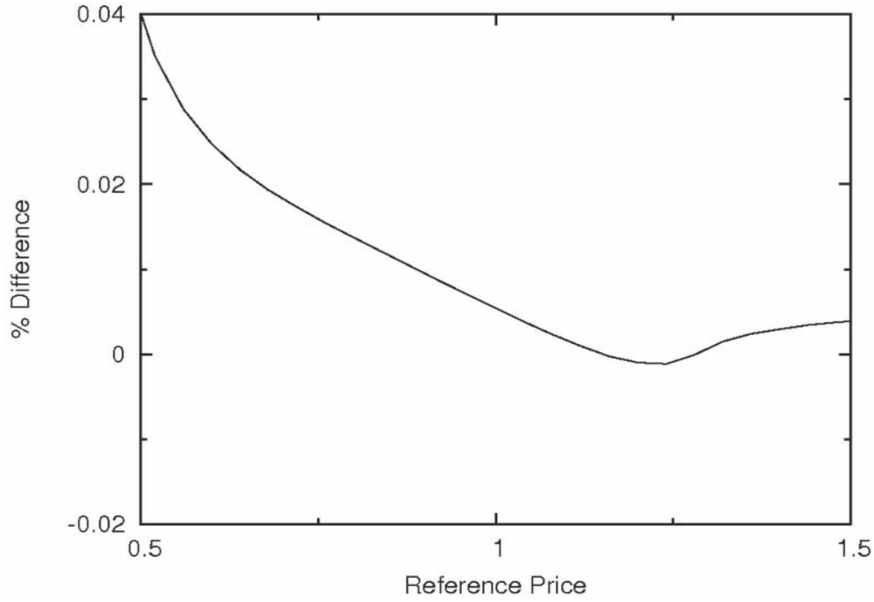

FIG 6 - This figure shows the effect of including ore grade uncertainty. The value shown is the percentage difference in net present value of the mine operation with no physical uncertainty minus the mine operation with physical uncertainty.

risks, in the presence of uncertainty surrounding their future cash-flows. To achieve this the RVOM uses advanced scientific methods to produce useful solutions in a fast time.

Although real options have long been considered as potentially extremely useful to mine operators, the inherent complexities and operational diversities have hindered the progress of mass-producing the necessary software. As a consequence, the full impact of a real-options approach to optimal mine extraction has been hard to estimate. However it is hoped that this paper has shown that an efficient and operationally realistic real options approach to mine operations now exists in the form of the RVOM, and the long talked about benefits can now be financially realised.

ACKNOWLEDGEMENTS

This paper was supported by the Engineering and Physical Sciences Research Council, United Kingdom, via the University of Manchester Knowledge Transfer Fund and the Center for Interdisciplinary Computational and Dynamical Analysis (CICADA). The data, research and results have the official approval of Gemcom Software International.

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