Involutions in the Automorphism Groups of Small Sporadic Simple Groups

Chris, Bates and Peter, Rowley and Taylor, Paul

2011

MIMS EPrint: **2011.67**

ISSN 1749-9097

Involutions in the Automorphism Groups of Small Sporadic Simple Groups

Chris Bates, Peter Rowley and Paul Taylor

August 1, 2011

Abstract

For each of fifteen of the sporadic finite simple groups we determine the suborbits of its automorphism group in its conjugation action upon its involutions. Representatives are obtained as words in standard generators.

1 Introduction

Groups permeate many areas of mathematics. Sometimes they have cameo roles, other times they are centre stage. Frequently it is involutions, elements of order two, that are in the spotlight. For instance, in the topological arena we have involutory maps on the 3-sphere in connection with the Smith Conjecture (see, for example, [2], [13], [15]). While in Banach spaces we encounter such things as involutive gradings and fixed points of involutions (see [6], [12]). In areas of algebra, such as associative rings and algebraic groups, there are many sightings of involutions (see [14], [16], [17]). Involutions can often have a considerable influence on the structure of the group to which they belong. Even their absence can be telling – witness the Feit Thompson theorem [7]. For a finite group of even order, Brauer and Fowler [4] establish many results concerning involutions and other properties of the group. For example they bound the index of a proper normal subgroup in terms of the number of involutions the group possesses. In a similar vein, for a finite group with at least two conjugacy classes of involutions the Thompson order formula ([10], Theorem 35.1) gives its order using data closely associated with the involutions. In the case when we have a finite non-abelian simple group, more often than not, its involutions play a dominant role (see, for example, [11]).

This paper studies the involutions in $Aut(K)$ where K is a small sporadic finite simple group. By small we mean that K is isomorphic to one of the following groups:

$$M_{11}, M_{12}, J_1, M_{22}, J_2, M_{23}, HS, J_3, M_{24}, McL, He, Ru, Suz, Co_3, Co_2.$$

The diminutive appellation aligns, more or less, with said group having a non-trivial permutation representation of degree at most 6156. Several of the larger sporadic groups have been studied individually in [1], [18], [19] and [20]. So for the remainder of this paper K is assumed to be a small sporadic simple group and G is a subgroup of $Aut(K)$ containing K. Also t will denote an involution of G. Put $X = t^G$, the G-conjugacy class of t. Our aim is to study the suborbits of G in its conjugation action on X. Or, in other words, to determine the action of $C_G(t)$ on $X \setminus \{t\}$. This we do employing the services of the computational algebra packages GAP [8] and MAGMA [3] partnered by the electronic ATLAS [22]. It goes without saying therefore that we use the ATLAS notation and conventions as given in [5].

As our starting point we take the smallest non-trivial permutation representation of G as described in [22] with K being generated by standard generators denoted here, as in [22], by a and b. In the case when $G \neq K$, the standard generators for G are, again as in [22], denoted by c and d. Having chosen a suitable element in X to play the role of t, we then forage for elements $g_1, g_2, \ldots, g_k \in G$ so as $\{t, t^{g_1}, \ldots, t^{g_k}\}$ is a complete set of representatives for the $C_G(t)$-orbits of X. In doing this we make frequent use of the standard command `IsConjugate(H,x,y)` – this works effectively here since the degree of the permutation representation of G is no more than 6156. Also observe that we may write our conjugating element g_i as a word in a and b. Thus, our aim is to find g_i which have relatively 'small' length relative to the generating set $\{a, b\}$ for K. However we may not always achieve the minimum possible length. In more detail we proceed as follows, define $D_0(a, b) = \{1\}$ and for $j \in \mathbb{N}$

$$D_j(a, b) = \{\, xa, xb \mid x \in D_{(j-1)}(a, b)\}.$$

For $n \in \mathbb{N}$ set

$$D_{[n]}(a, b) = \bigcup_{j \leq n} D_j(a, b).$$

The main purpose of $D_{[n]}(a, b)$ is to produce a colony of short words in a and b, a number of which may well yield the same element of G. To speed matters up we prune out these duplicates. We hunt through $D_{[n]}(a, b)$ for n typically at most 20, so as to ensnare suitable conjugating elements g_i. If this does not yield enough representatives for the $C_G(t)$-orbits, then we recalculate $D_{[n]}(a, b)$ with a and b replaced by short words in a and b which are also generators for G. For example we might try replacing a by ab and keeping b the same, or try a more complicated substitution such as replacing b with $(ab)^5a$. Such a substitution was used to produce a g_i in the case when $G \cong M_{24}$ and tx in the class $3A$, starting from the word bab^2abab^2ab. Some of the words for the g_i may be further simplified and this was done by hand.

On a number of occasions this approach fails to deliver conjugating elements for some (often of small size) $C_G(t)$-orbits. To deal with such elusive

$C_G(t)$-orbits, say $\mathcal{O}$, we begin by finding an x in $\mathcal{O}$, usually by a random search. Then we obtain an element h in G such that $t^h = x$. Now we run through some or all of the elements c in $C_G(t)$, seeking a c for which ch has small 'length' (that is, the total number of symbols a, a^{-1}, b, b^{-1} in the given expression for ch). Sometimes we may also vary h. In somewhat more detail the MAGMA code employed is as follows.

```
F<a,b>:=FreeGroup(2);
phi:=hom< F->G | <a,G.1>,<b,G.2> >;

r,h:=IsConjugate(G,t,x);
sh:=h@@phi;
lengthmin:=#sh;
hmin:=h;
Ct:=Centralizer(G,t);

for c in Ct do
   sch:=(c*h)@@phi;
     if #sch lt lengthmin then
       lengthmin:=#(sch);
       hmin:=c*h; shmin:=sch;
   end if;
end for;
```

We remark that this procedure is not usually as efficient as using the set $D_{[n]}(a, b)$ and moreover usually yields more complicated expressions for the conjugating elements.

One may envision the information presented here to be useful in the following circumstances. Suppose H is a finite group with $x, y \in H$. Further suppose that we have identified the subgroup $L = \langle x, y \rangle$ as being isomorphic to G via recognizing x and y as standard generators. (See [21] for a discussion of standard generators.) Then we may translate our information from G to L so as we see how L acts upon its involution conjugacy classes within the group H. Moreover, if H is large from a computational standpoint (for example $|H|$ is large or H is a matrix group of large degree), then having the g_i as short words may be beneficial. To facilitate applications such as these, computer files containing the g_i as words in a and b are available on request from the second author.

The following section gives the permutation ranks of G on X and the elements g_i together with some additional information.

2 Orbit Representatives

So t is a suitably chosen (and then fixed) involution of the conjugacy class X of G. For C a G-conjugacy class we define

$$X_C = \{\, x \in X \mid tx \in C\}.$$

Since X_C is $C_G(t)$-invariant, X_C will be a union of $C_G(t)$-orbits for each G-conjugacy class C for which $X_C \neq \emptyset$. As is well-known, $|X_C|$ may be calculated for any G-conjugacy class using the complex character table (see, for example, [9]), and this is easily carried out in GAP. As we proceed by breaking each X_C (for $|X_C| \neq 0$) into $C_G(t)$-orbits this is useful information.

Our first table gives an overview of the permutation ranks of G in its action on X. The succeeding tables consider in turn the possibilities for K, with the first column identifying X_C, the second the size of the $C_G(t)$-orbit $\mathcal{O}$ (contained in X_C) and the third supplies a group element g for which $t^g \in \mathcal{O}$.

We emphasize that the following tables give the $C_K(t)$-orbits of X – in the case when $K < G = \mathrm{Aut}(K)$ the tables are annotated so as to also yield the $C_G(t)$-orbits of X. Before explaining how this is done, we remark that in all instances here when $K < G = \mathrm{Aut}(K)$ we have $C_K(t) < C_G(t)$. Thus X is a G-conjugacy class and $G = KC_G(t)$. Now suppose that C_1 and C_2 are two K-conjugacy classes which fuse in G (so $C_1 \cup C_2$ is a G-conjugacy class). Then $C_1^g = C_2$ for some $g \in C_G(t)$ and consequently $X_{C_1}^g = X_{C_2}$. Hence, if $\mathcal{O}$ is a $C_K(t)$-orbit contained in X_{C_1}, $\mathcal{O} \cup \mathcal{O}^g$ will be a $C_G(t)$-orbit of X. In this circumstance a broken horizontal line indicates that C_1 and C_2 are fused in $G = \mathrm{Aut}(K)$ and if $\mathcal{O}$ is obtained by using the ith listed conjugating element in X_{C_1} then $\mathcal{O}^g$ is obtained using the ith listed conjugating element in X_{C_2}. When a K-conjugacy class C is also a G-conjugacy class, it may be the case that a $C_G(t)$-orbit contained in X_C is the union of two $C_K(t)$-orbits in X_C. A vertical line connecting two $C_K(t)$-orbits in X_C signifies that their union is a $C_G(t)$-orbit.

2.1 Permutation Ranks

G	$2A$	$2B$	$2C$	$2D$
M_{11}	8			
M_{12}	10	11	13	
J_1	22			
M_{22}	13	5	10	
J_2	6	31	14	
M_{23}	9			
HS	9	27	5	32
J_3	27	17		
M_{24}	14	26		
McL	6	28		
He	12	34	41	
Ru	22	42		
Suz	9	66	15	41
Co_3	7	61		
Co_2	5	13	178	

2.2 $K \cong M_{11}$

2.2.1 $X = 2A, t = a$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	12	bab^3ab
$3A$	24	b^2
	8	b^3ab
$4A$	24	bab
$5A$	48	$babab$
$6A$	24	b
	24	b^3

2.3 $K \cong M_{12}$

2.3.1 $X = 2A, t = ((ab)^2ab^2)^3$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	10	b^2abab^2abab
	10	$b^2ab^2abab^2ab$
$2B$	15	a
$3B$	60	ab
$4A$	30	ab^2aba
$4B$	30	$bababab^2a$
$5A$	120	ba
$6A$	60	b
	60	b^2

2.3.2 $X = 2B, t = a$

X_C	$\lvert\mathcal{O}\rvert$	g
$2B$	24	bab^2ab
	6	$b^2ababab^2ababab^2abab$
$3A$	32	b^2ab
	32	bab^2
$3B$	16	bab^2ab^2abab
$4A$	48	$babab^2$
$4B$	48	b^2abab
$5A$	96	bab
$6B$	96	b
	96	b^2

2.3.3 $X = 2C, t = c$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	15	$bab^2ababab$
	1	$bababab^2ab^2abab^2ab$
$2B$	15	$babab^2$
$3A$	20	$b^2ababab$
	20	$abab^2aba$
$3B$	60	bab^2
$5A$	60	$abab^2a$
	60	ab
$6A$	60	$baba$
$6B$	60	b^2ab
	60	a
$10A$	60	bab^2aba
	60	$babab$
$11A$	120	b
$11B$	120	ba

2.4 $K \cong J_1$

2.4.1 $X = 2A, t = a$

X_C	$\|\mathcal{O}\|$	g
$2A$	15	$bab^2abab^2abab^2abab^2abab$
	15	$bab^2ab^2abab^2ababab^2abab$
$3A$	60	$bab^2ababab^2ab$
	12	$b^2ab^2abab^2abab^2abab$
$5A$	60	$babab^2ab^2abab$
	20	$b^2ababab^2ab^2ab$
$5B$	60	$babab^2abab^2abab$
	20	b^2abab^2abab
$6A$	60	bab^2abab
	60	$babab^2ab$
$7A$	120	bab^2ab^2abab
$10A$	60	$b^2ababab$
	60	$bababab^2$
$10B$	60	b^2abab
	60	$babab^2$
$11A$	120	$babab^2abab$
$15A$	120	b^2ab^2abab
$15B$	120	bab
$19A$	120	b
$19B$	120	b^2ab
$19C$	120	b^2abab^2ab

2.5 $K \cong M_{22}$

2.5.1 $X = 2A$, $t = a$

X_C	$\|\mathcal{O}\|$	g
$2A$	24	bab^3ab
	12	$(ba)^2b^2abab$
	8	$b^2abab^2(ab)^3$
	6	$b^3(ab)^2(ab^3)^2ab$
$3A$	192	bab
$4A$	48	bab^2ab
	48	b^3abab
	48	$babab^3$
$4B$	96	b^2abab
	96	$babab^2$
$5A$	384	b^2
$6A$	192	b

2.5.2 $X = 2B$, $t = c$

X_C	$\|\mathcal{O}\|$	g
$2A$	42	b
	7	$(b^2a)^2ba$
$3A$	112	a
$4A$	168	$ababa$

2.5.3 $X = 2C$, $t = (cdcd^2)^5$

X_C	$\|\mathcal{O}\|$	g
$2A$	20	b^2
	5	bab^2ab^3
$3A$	80	b
$4A$	80	$bababa$
$4B$	40	bab^2a
	40	bab^2ab
$5A$	320	ab
$6A$	80	a
	80	ab^2a
$11A$	320	b^2a
$11B$	320	ba

2.6 $K \cong J_2$

2.6.1 $X = 2A, t = (bab^2(ab)^2b(ab)^3)^5$

X_C	$\|\mathcal{O}\|$	g
$2A$	10	ba
$3B$	160	b
$4A$	80	a
$5A$	32	ab^2aba
$5B$	32	bab^2

2.6.2 $X = 2B, t = a$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	15	$b(ba)^2b(ba)^2b(ba)^2$
$2B$	15	$((ba)^2b)^3(ba)^3$
	15	$b(ba)^2(b(ba)^3b^2a)^2$
	1	$(ba)^2b^2a(b(ba)^2)^2b^2ab(ba)^3$
	1	$b^2ab(ba)^3b(ba)^2b(ba)^3(b^2a)^2$
$3A$	24	$b(ba)^2b(ba)^2$
$3B$	60	$b(ba)^3b(ba)^2b(ba)^3b(ba)$
$4A$	60	$(ba)b(ba)^2$
$5A$	60	$(ba)^2b(ba)^2b(ba)^3b(ba)b(ba)^2$
$5B$	60	$(ba)b(ba)b(ba)^3b(ba)^2b(ba)^3$
$5C$	120	$b(ba)^3b(ba)^3$
	24	$b(ba)b(ba)^2b(ba)^3$
$5D$	120	$(ba)b(ba)^2b(ba)^3$
	24	$(ba)^2b(ba)^2b(ba)b(ba)$
$6A$	120	$(ba)^3$
$6B$	60	$(ba)^2b(ba)^3$
	60	$(ba)^3b(ba)^4$
$7A$	240	$b(ba)^4$
$10A$	60	$b(ba)b(ba)^2b(ba)^2$
	60	$b(ba)^2b(ba)^3$
$10B$	60	$(ba)^2b(ba)^2b(ba)$
	60	$(ba)b(ba)^2b(ba)b(ba)$
$10C$	120	$(ba)b(ba)b(ba)$
	120	$(ba)^2b(ba)$
$10D$	120	$b(ba)^3$
	120	$b(ba)b(ba)^2$
$12A$	120	$b(ba)$
	120	(ba)
$15A$	240	$(ba)b(ba)b(ba)^2b(ba)^2$
$15B$	240	$(ba)b(ba)^2b(ba)b(ba)^2$

2.6.3 $X = 2C, t = c$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	21	$(bab^2a)^2(ba)^3$
$2B$	28	bab^2
$3A$	14	$(ab)^3(ab^2)^2ab$
	14	$(b^2aba)^2bab$
$3B$	168	$baba$
$4A$	84	$abab^2ab$
	42	b^2abab
$6A$	84	b^2
	84	b
	42	$abababa$
	42	$abab^2abab$
$6B$	168	ab
$7A$	336	bab
$8A$	168	ab^2a
	168	aba
$12A$	168	ba
	168	a

2.7 $K \cong M_{23}$

2.7.1 $X = 2A, t = a$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	84	bab^3ab
	14	b^3ab^2ab
$3A$	448	b^2
$4A$	336	$babab$
	336	$bababab$
	336	$bababab^2$
$5A$	896	bab
$6A$	1344	b

2.8 $K \cong HS$

2.8.1 $X = 2A, t = a$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	80	$b^2ab^3ab^2$
	30	$b^2abab^2abab^2$
$3A$	640	bab
$4B$	480	bab^3ab
$4C$	960	b^2ab
$5B$	128	bab^2ab
$5C$	1536	b
$6B$	1920	b^2

2.8.2 $X = 2B, t = ((ab)^2a)^5$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	45	bab^3ab
	30	b^4ab^4abab
$2B$	36	$b^2ab^3ab^2$
	36	$bababab^3a$
$3A$	360	b^3ab^2abab
	24	$ab^2ab^2abab^2ab^2a$
$4B$	180	a
$4C$	360	$babab^4ab^2a$
	180	$bababab^2$
	180	$b^2ababab$
$5A$	144	bab^4ab^3ab
	144	bab^3ab^4ab
$5B$	720	$babab^2a$

X_C	$\lvert\mathcal{O}\rvert$	g
$6B$	720	bab
	360	bab^2a
	360	ab^2ab
$7A$	2880	b
$8B$	720	b^4ab^2a
	720	ab^3ab
$8C$	720	bab^3a
	720	ab^2ab^4
$10A$	720	b^2
	720	b^3
$10B$	720	aba
	720	ba
$15A$	2880	b^2a

2.8.3 $X = 2C, t = c$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	105	a
$2B$	28	ba
$3A$	336	b^2
$4B$	630	b

2.8.4 $X = 2D, t = ((dc)^2d^4cd)^5$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	60	ab^3abab^2a
	5	ab^3ab^2a
$2B$	60	b^2a
$3A$	240	a
	160	ab^3abab^2aba
$4A$	120	bab^3aba
$4B$	240	ab
	120	bab^2abab
	30	ab^2a
$4C$	120	ab^2ab^2a
	120	ab^3aba
$5A$	192	$abab^3$
	192	bab^2ab^3
$5B$	320	b^4ab
$5C$	1920	b^2ab^2
$6A$	480	ab^2ab
	480	bab^2a

X_C	$\lvert\mathcal{O}\rvert$	g
$6B$	480	b^4abab
	480	b^2ab^2a
$7A$	1920	b
$8A$	480	$babab^2$
	480	bab^2ab^2ab
$10A$	960	$b^2ab^2ab^2a$
	960	bab
$10B$	960	ab^2
$11A$	1920	b^2aba
$11B$	1920	$babab$
$12A$	960	ba
	960	b^3
$15A$	1920	bab^3
$20A$	960	$baba$
	960	b^2ab
$20B$	960	b^2ab^3a
	960	b^4aba

2.9 $K \cong J_3$

2.9.1 $X = 2A, t = a$

X_C	$\|\mathcal{O}\|$	g
$2A$	120	$b(ba)^3b(ba)b(ba)^5$
	10	$(ba)b(ba)^4b(ba)^2$
$3A$	192	$b(ba)^6$
$3B$	640	$(ba)b(ba)^2$
$4A$	480	$b(ba)^5b(ba)$
	80	$(ba)^3b(ba)^3b(ba)^4$
$5A$	960	$(ba)^3b(ba)^3b(ba)^2$
	320	$(ba)^2b(ba)^2$
$5B$	960	$(ba)b(ba)^3b(ba)^4$
	320	$(ba)b(ba)^3$
$6A$	960	$(ba)b(ba)^6$
$8A$	960	$(ba)^4b(ba)^2$
	960	$(ba)b(ba)^5$
$9A$	1920	$b(ba)^2b(ba)^2$
$9B$	1920	(ba)
$9C$	1920	$b(ba)^2$
$10A$	960	$(ba)^4b(ba)$
	960	$b(ba)^4b(ba)b(ba)$
$10B$	960	$b(ba)b(ba)b(ba)b(ba)^2$
	960	$b(ba)^5$
$12A$	960	$(ba)^2b(ba)^4$
	960	$(ba)^6$
$15A$	1920	$(ba)^2b(ba)$
$15B$	1920	$b(ba)^3$
$17A$	1920	$(ba)^2$
$17B$	1920	$(ba)^2b(ba)^3$

2.9.2 $X = 2B, t = c$

X_C	$\|\mathcal{O}\|$	g
$2A$	153	$ab^2ababab$
$3A$	102	b^2
	102	b
$3B$	272	$abab^2a$
$4A$	612	$bababab^2a$
	306	bab^2ababa
$6A$	612	bab^2aba
	612	$babab^2a$
	306	ab^2ab^2ababa
	306	$baba$
$8A$	1224	$ababab$
	1224	b^2ababa
$9A$	816	b^2abab^2ab
$9B$	816	$babab^2ab$
$9C$	816	bab^2
$12A$	1224	ab
	1224	ab^2aba
$17A$	2448	ba
$17B$	2448	a
$19A$	2448	ab^2ab
$19B$	2448	b^2aba

2.10 $K \cong M_{24}$

2.10.1 $X = 2A, t = ((ab)^3ab^2)^4$

X_C	$\|\mathcal{O}\|$	g
$2A$	168	$b^2ababab^2ab$
	14	aba
	14	$abab^2ab$
$2B$	84	b^2ab
$3A$	896	b
$3B$	128	$b^2ab^2abab^2abab$

X_C	$\|\mathcal{O}\|$	g
$4A$	112	$bababab$
	112	$b^2ab^2ab^2ab^2$
$4B$	1344	ab^2aba
	672	ab^2abab^2ab
	672	b^2abab^2aba
$5A$	1792	$baba$
$6A$	5376	a

2.10.2 $X = 2B, t = a$

X_C	$\|\mathcal{O}\|$	g
$2A$	60	$b^2abab^2ab^2abab^2abababab^2ab$
	15	$bab^2ab^2abababab^2$
$2B$	120	$bab^2abab^2abab^2$
	80	$b^2abab^2abab^2ab$
	2	$(ba)^8(b^2a)^2bab^2(ab)^2(ab^2)^2(ab)^3(ab^2ab)^2b$
$3A$	320	$b^2ab^2abab^2ababab$
$3B$	960	b^2abab^2ab
$4A$	120	bab^2ab^2
	120	$babab^2$
$4B$	480	$b^2ababab^2ab$
	240	$bab^2ababab^2abab^2ab$
	240	$b^2abababababab$
$4C$	960	bab^2ab
	960	b^2abab^2
	160	$bababab^2abababab^2ab^2$
	160	$b^2abababab^2ab^2ababab$
$5A$	1920	bab
$6A$	1920	$bababab^2$
$6B$	1920	bab^2
	1920	b^2ab
$10A$	1920	b^2abab^2abab
	1920	$b^2abababab^2$
$11A$	7680	$babab$
$12B$	3840	b
	3840	b^2

2.11 $K \cong McL$

2.11.1 $X = 2A, t = a$

X_C	$\|\mathcal{O}\|$	g
$2A$	210	$b^2ab^3ab^2$
$3B$	2240	$babab^2$
$4A$	5040	b^2ab
$5B$	8064	b
$6B$	6720	b^2

2.11.2 $X = 2B, t = c$

X_C	$\|\mathcal{O}\|$	g
$2A$	165	ab^2abab^2ab
$3A$	220	$b^2ab^3ab^2abab^2a$
$3B$	660	ab^2ab^2
	220	$b^2abababab^3$
$4A$	1980	b^2ab^3ababa
	990	a
$5A$	792	$b^2ab^2ab^3$
	792	$b^2abababab^2$
$5B$	7920	b^2
$6A$	1980	ab
$6B$	1980	bab^2abab
	1980	ab^2ab
	660 |	$abab^3$
	660 |	bab^3ab^4ab
$7A$	3960	$babab^2a$
	3960	bab^2
$7B$	3960	bab^3a
	3960	b^2a
$8A$	3960	b^2ab^2a
	3960	ab^3ab^3
$9A$	2640	b^2aba
$9B$	2640	$abab^2ab$
$10A$	3960	$babab$
	3960	$ababa$
$12A$	3960 |	$babab^3$
	3960 |	bab^3
$14A$	3960	ab^2ab^3
	3960	bab^2ab
$14B$	3960	$bababa$
	3960	b^2ababa
$15A$	3960	bab
	3960	b^4
$15B$	3960	$abab$
	3960	b
$30A$	3960	ba
	3960	bab^2a
$30B$	3960	b^4ab^2a
	3960	ab^2

2.12 $K \cong He$

2.12.1 $X = 2A, t = a$

X_C	$|\mathcal{O}|$	g
$2A$	240	$bababab$
	2	$b^5(ab^6)^2(ab)^3b^5a(b^2ab^3)^2$
$2B$	315	$b^2abab^2ab^2ab$
$3A$	1344	b^2ab
$3B$	960	bab^4ab
$4A$	480	b^3abab^2ab
	480	b^2abab^3ab
$4B$	2520	b^2abab
	2520	b^4ab
$5A$	2688	b^3
$6A$	13440	b

2.12.2 $X = 2B, t = (ab^2)^6$

X_C	$|\mathcal{O}|$	g
X_C	$|\mathcal{O}|$	g
$2A$	84	$(ab)^2b^5(ab)^2$
$2B$	168	$(ab)^2$
	168	$(ab)b^4(ab)b^2(ab)b^2(ab)b^4$
	14	$b^2(ab)^2b^4(ab)b^4(ab)b^2(ab)b^2$
	14	$b^2(ab)b^2(ab)^2b^4(ab)b^2(ab)b^2$
$3A$	896	$b^4(ab)b^2$
$3B$	2688	b^2
$4B$	1344	$b^2(ab)b^2(ab)b^2(ab)b^2$
	1344	$b^2(ab)b^4(ab)^2b^4$
	112	$(ab)b^2(ab)^4b^4(ab)^2$
	112	$(ab)^3b^4(ab)^4b^2$
$4C$	1344	$(ab)b^4(ab)b^4$
	1344	$b^4(ab)b^2(ab)b^2(ab)^3b^2$
	672	$b^2(ab)^2b^2(ab)^3$
	672	$b^2(ab)^4b^2(ab)^2$
	672	$b^2(ab)b^2(ab)^4b^2(ab)^2$
	672	$(ab)^4b(ab)^2$
$5A$	5376	$(ab)^3$
$6A$	5376	$(ab)b^2(ab)b^2(ab)^4$
$6B$	5376	$b^2(ab)^2b^2(ab)$
	5376	$(ab)b^2(ab)$
$7C$	3072	$b^2(ab)b^2(ab)b^2(ab)^2$

... continued on next page

... continued from previous page

X_C	$\|\mathcal{O}\|$	g
$8A$	5376	$(ab)^2b^2(ab)^2b^2$
	5376	$(ab)b^2(ab)^2b^2(ab)$
$10A$	5376	$b^2(ab)^2$
	5376	$b^2(ab)b^2(ab)b^2$
$12B$	10752	$b^2(ab)b^2(ab)^2$
	10752	$b^4(ab)^2$
$15A$	21504	$b^2(ab)^3$
$17A$	21504	$(ab)b^2(ab)^2$
$17B$	21504	(ab)
$21A$	21504	$(ab)b^2(ab)b^2$
$21B$	21504	$b^2(ab)$

2.12.3 $X = 2C, t = d^3$

X_C	$\|\mathcal{O}\|$	g
$2A$	105	$(ab)b(ab)^3b(ab)^3$
	63	$b^6ab^3ab^3ab^2ab^6ab^2ab^6ab^6ab^4a$
$2B$	315	$(ab^2)^2(ba)^2b^2a$
$3A$	630	$(ab)b(ab)b(ab)b$
	126	$b(ab)^2b^6$
$3B$	2520	$(ab)b^6$
$4A$	1260	$(ab)^2b^5(ab)$
	1260	$(ab)b(ab)b^3$
$4B$	945	$(ab)^2b^2(ab)b$
	945	$b(ab)b(ab)b^2(ab)b$
	315	$b(ab)^5b$
	315	$b(ab)^3b^2(ab)b$
$4C$	1890	$b^3(ab)^2$
	1890	$(ab)b^2(ab)^2b$
$5A$	2520	$(ab)^3b(ab)$
$6A$	3780	$(ab)^3b^2(ab)^2$
	1890	$b(ab)b(ab)$
	1890	$b(ab)b(ab)b(ab)^3$
	1260	$b(ab)b^2(ab)b(ab)^2$
$6B$	7560	(ab)
$7A$	1890	$(ab)b(ab)^4b^2$
	90	$b(ab)b^3(ab)^2b$

... continued on next page

... *continued from previous page*

X_C	$\lvert\mathcal{O}\rvert$	g
$7B$	1890	$b^5(ab)b(ab)$
	90	$b^5(ab)b^5(ab)b^2(ab)b^5$
$7C$	2160	$b(ab)b(ab)^4$
$7D$	7560	$(ab)b(ab)^4$
	1080	$(ab)b^3(ab)$
$7E$	7560	$(ab)^3b(ab)^2$
	1080	$(ab)b^4(ab)^2b(ab)$
$8A$	3780 |	$(ab)^2b(ab)^2$
	3780 |	$(ab)^4$
	3780 |	$(ab)^3b(ab)b(ab)b$
	3780 |	$(ab)b(ab)b(ab)^2$
$10A$	7560	$(ab)^3$
$12A$	7560	$b(ab)b$
	7560	$b(ab)b^2$
$12B$	7560 |	$b(ab)^2b$
	7560 |	b
$14A$	3780	$(ab)b^3(ab)^2$
	3780	$(ab)^2b^2(ab)$
	1890	$(ab)b^2(ab)$
	1890	$b(ab)^3b$
$14B$	3780	$(ab)b(ab)^2b(ab)$
	3780	$(ab)b^4$
	1890	$(ab)^2b^2(ab)b(ab)^2$
	1890	$b(ab)b(ab)^2$
$14C$	7560	$b(ab)$
	7560	$b(ab)^3$
$14D$	7560	$b^3(ab)$
	7560	$b(ab)^2$
$15A$	15120	$(ab)b^2$
$21A$	15120	$(ab)^4b(ab)$
$21B$	15120	$(ab)b$
$21C$	15120	$(ab)^3b$
$21D$	15120	b^2
$28A$	7560	$b^2(ab)b^2$
	7560	$(ab)^2$
$28B$	7560	$b^4(ab)^3$
	7560	$(ab)b(ab)^2$

2.13 $K \cong Ru$

2.13.1 $X = 2A, t = b^2$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	640	ab^3ab^2aba
	480	$ab^2ab^2abababab^2ab^2a$
	240	$ab^2ab^3ab^3ab^3ab^3ab^2abab^2ab^3ababab^3ab^3$
	30	$ab^2ababab^2ab^2ab^3ab^3ababab^2ab^3ab^2ab^2ab^3ab^2ababab^3a$
$3A$	10240	$ab^3abababab^2a$
$4A$	5120	$ababab$
$4C$	7680	$abab^2ab^2ab^2aba$
	3840	$ab^2ab^3abab^2aba$
	3840	$abab^2abab^3ab^2a$
$4D$	15360	aba
	7680	$ab^2ab^2abab^2abab^2a$
$5A$	12288	ab^3abab^2ababa
$5B$	4096	$ababa$
$6A$	61440	ab^2abab^2aba
	30720	$ababababa$
$7A$	61440	a
$8C$	61440	$abab^2ab^2a$
	61440	ab^2ab^2aba
$10A$	61440	ab^2ab^2ababa
$12A$	122880	ab^2aba
$13A$	61440	ab^3aba

2.13.2 $X = 2B, t = a$

X_C	$\lvert\mathcal{O}\rvert$	g
X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	455	$bab^3ab^2ab^3ab$
$2B$	455	$bab^3ab^2abababab$
	455	$babababab^2ab^3ab$
	1	$b^3ab^3ab^3ab^2ababab^2ab^2ab^3abab^2ab^2ab^2$
	1	$bab^2abab^3abab^2ab^2abab^2abab^3ab^3ababab^3abab^2abab^2ab^3abab^3$
$3A$	5824	$b^2abababab$
$4A$	3640	$(ab)^{10}a(ab)^{15}a((ab)^5a)^2$
$4C$	3640	$b^3ababab^2ab^2$
$4D$	3640	$b^3abab^2ab^3ab$
$5A$	5824	$b^3abab^2abab^2$
	5824	$b^2ab^3ab^2ab$
$5B$	29120	b^2abab

... continued on next page

... continued from previous page

X_C	$\|\mathcal{O}\|$	g
$6A$	29120	$b^2ababab^2ab$
$7A$	29120	b^2ab^2
	29120	b
	29120	b^2
$10A$	29120	$babab^2abab$
	29120	$babab$
$10B$	29120	$babab^3ab^3ab$
	29120	bab^3ab^3abab
$12A$	58240	$b^3ababab^2ab$
$13A$	29120	bab
	29120	$bababab$
	29120	b^3ab^2ab
$14A$	29120	$b^3ab^2ab^2abab$
	29120	$bab^2ab^2ababab$
$14B$	29120	$b^2ab^2ababab$
	29120	$bababab^2ab^2$
$14C$	29120	$bab^2abababab^2$
	29120	$b^2abababab^2ab$
$15A$	116480	$b^3ab^2ababab$
$20A$	58240	b^3abab^2
	58240	b^2ab^3ab
$26A$	29120	bab^2
	29120	b^2ab
$26B$	29120	bab^2ab^2
	29120	b^2ab^2ab
$26C$	29120	$b^3ababab$
	29120	$babab^2ab^2$
$29A$	116480	bab^2ab
$29B$	116480	b^3abab

2.14 $K \cong Suz$

2.14.1 $X = 2A, t = ((ab)^2ab^2)^6$

X_C	$\|\mathcal{O}\|$	g
$2A$	360	a
	54	$((ab)^2a)^4a((ab)^2a)^3$
$3B$	5120	$b^2ab^2abababababa$
$3C$	9216	$baba$
$4A$	1728	$b^2ab^2ab^2ababab$
$4C$	17280	b^2ab
$5B$	55296	b^2ababa
$6D$	46080	b

2.14.2 $X = 2B, t = a$

X_C	$\|\mathcal{O}\|$	g
$2A$	315	$bab^2ababab^2ab^2ab^2ab^2ab^2abababab^2ab$
$2B$	630	$b^2abababab^2ababab^2ab^2ababab^2ab^2abab^2ab^2ab^2ab$
	560	$bab^2abab^2abab^2abab$
	2	$(bab^2a)^2(b^2a)^2(ba(b^2a)^2)^2(b^2aba)^2((b^2a)^2b^2aba)^2bab^2(ab)^2$
$3A$	224	$(bab^2a)^3b(ba)^3(b^2a)^2(ba)^5b(ba)^4(b^2a)^2b$
$3B$	2240	$b^2ab^2ababab^2abab$
$3C$	20160	bab^2abab^2ab
$4A$	2520	$bababab^2ab^2ab^2ab^2ababab$
	1260	$b^2ababab^2ababab^2abab$
$4B$	2520	$b^2ab^2ababab^2abab^2ab^2abab$
	2520	$bab^2ab^2ababab^2abababab$
	1260	$b^2ababab^2ab^2ab$
	1260	$b^2ab^2abab^2abab$
$4C$	5040	$bab^2abab^2ababab^2abab^2ab$
	2520	$bababab^2ab^2ababab$
	2520	$b^2ab^2ababab^2abab^2ab^2abab$
$4D$	10080	$bab^2ab^2ab^2abab^2ab^2abab$
	10080	$babab^2ab^2abab^2ab^2ab^2ab$
	1120	$(ab)^4((ab)^4a)^4a(ab)^8((ab)^4a)^2$
	1120	$(a(b^2a)^4)^2(b^2a)^8a(a(b^2a)^4)^4(b^2a)^4$
$5A$	20160	$babab$
$5B$	40320	$b^2ababab^2ab$
$6A$	10080	$babababab^2abababab$
$6D$	20160	bab^2ab^2ab
$6E$	40320	$b^2ababab^2ababab$
	40320	$b^2abab^2ab^2ab^2ababab$

... continued on next page

... continued from previous page

X_C	$\lvert\mathcal{O}\rvert$	g
$7A$	80640	$b^2abababab$
$8A$	20160	$bab^2abab^2ab^2ab^2ab$
	20160	$bab^2ab^2ab^2abab^2ab$
$8B$	20160	$bab^2abababab^2ab^2abab^2ab$
	20160	$babab^2ab^2ab$
	20160	bab^2ab^2abab
	20160	$bab^2abab^2ab^2abababab^2ab$
	10080	b^2ab^2ab
	10080	b^2abab
$8C$	20160	$b^2abab^2ababab$
	20160	$b^2ababab^2abababab^2ab$
	20160	$b^2ab^2ab^2abab^2ab$
	20160	$bab^2ab^2abab^2ab^2ababab$
$10A$	40320	$b^2abab^2abab^2abab$
	40320	$abababab^2ababab$
	20160	$b^2ab^2ababab^2ababab$
	20160	$b^2ab^2ab^2abab^2ab^2abab$
$10B$	40320	$babababab^2ab$
	40320	$bab^2abababab$
$11A$	161280	b^2abab^2abab
$12A$	40320	$b^2ab^2abab^2abab^2abab$
	20160	$b^2abab^2ab^2ababab^2ab$
$12B$	40320	$b^2ababababab$
	40320	$b^2ab^2ab^2ab^2ab^2ab$
$12C$	40320	$bab^2abab^2ab^2ababab$
$12D$	80640	$bab^2ababab$
	80640	$bababab^2ab$
$13A$	161280	bab
$13B$	161280	b^2ab^2abab
$14A$	80640	$b^2abab^2ab^2abab$
$15A$	161280	b
$15B$	161280	b^2ab
$15C$	161280	$b^2ab^2abab^2ababab$
$20A$	80640	$bab^2ab^2ab^2abab$
	80640	$babab^2ab^2ab^2ab$
$21A$	161280	$bab^2ab^2ab^2ababab$
$21B$	161280	$bab^2ab^2ab^2ab$
$24A$	80640	$babab^2ab$
	80640	bab^2abab

2.14.3 $X = 2C, t = c$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	315	$((ab)^3a)^2a((ab)^3a)^4a(ab)^3((ab)^3a)^2a$
$2B$	1800	$ababab^2$
$3A$	200	$b^2ab^2abab^2abab$
$3C$	16800	ab^2ab^2ababa
$4A$	3150	bab^2abab^2a
	630	$bab^2abababababab^2a$
$4C$	12600	bab^2aba
$5B$	20160	aba
$6A$	12600	$ababab^2ab$
$6E$	100800	ba
$7A$	100800	b
$8A$	25200	$ababab$
	25200	$bababab$
$12A$	50400	a

2.14.4 $X = 2D, t = (c(cd)^{12})^{11}$

X_C	$\lvert\mathcal{O}\rvert$	g
$2A$	495	$((ab)^5)a)^2a(ab)^5)((ab)^5)a)^5a(ab)^5)$
$2B$	792	$b^2abab^2ababab^2abab$
$3A$	264	$a(ab)^9((ab)^9a)^4a(ab)^9((ab)^9a)^2$
$3B$	5280	$a(ab)^6((ab)3a)^4$
	1760	$(a((ab)^3a)^2)^4a$
$3C$	2640	$b^2ababab^2ababab^2a$
$4A$	2970	$babab^2abab^2aba$
	495	$b^2abababab$
	495	$bababab^2ababa$
$4B$	11880	$bab^2ab^2abab^2aba$
$4C$	11880	b^2ab
$5A$	19008	$baba$
$5B$	47520	ab^2ababa
$6A$	11880	aba
	3960	$ababab^2a$
	3960	$abab^2ab^2a$
$6B$	15840	ab^2abab
	15840	$ab^2abababab$
$6C$	15840	b^2ab^2aba
	15840	$b^2abababab^2ab^2$
$6D$	47520	ab^2ababa

. . . continued on next page

. . . continued from previous page

X_C	$\lvert\mathcal{O}\rvert$	g
	15840	$ab^2ababab^2aba$
$6E$	15840	$abababab^2abab^2ab^2ab$
$7A$	47520	bab^2aba
$8B$	47520	$b^2abababab a$
	47520	$ababab$
$9A$	95040	$bababab$
	31680	$babab^2ab^2abab^2a$
$9B$	95040	b^2ababa
	31680	$ab^2abababababa$
$10A$	95040	ab
$10B$	47520	$babab^2ab^2a$
	47520	bab^2ab^2aba
$11A$	190080	a
$12A$	47520	b^2aba
$12B$	47520	$abab^2ababab^2$
	47520	$abababab a$
	15840	b^2ab^2abab
	15840	$bababa$
$12C$	47520	ab^2aba
	47520	$ababababab ab^2$
$12E$	47520	b^2ab^2a
	47520	$abab$
	47520	$babababab$
	47520	$b^2abababa$
$14A$	47520	ab^2abab^2abab
	47520	b
$15C$	190080	$ababa$
$18A$	95040	bab^2ababa
	95040	ab^2ab^2abab
$18B$	95040	b^2ab^2ababa
	95040	$bababab^2a$
$20A$	95040	$babab^2aba$
	95040	ba

2.15 $K \cong Co_3$

2.15.1 $X = 2A, t = b^2$

X_C	$\|\mathcal{O}\|$	g
$2A$	630	$ab^2a^2b^2a$
$3B$	8960	ab^2a^2
$3C$	1920	a^2ba^2
$4B$	30240	ab^2a
$5B$	48384	aba^2
$6C$	80640	a

2.15.2 $X = 2B, t = (a^2b)^3ab)^5$

X_C	$\|\mathcal{O}\|$	g
$2A$	495	$baba^2ba^2bab^2ab$
$2B$	396	$ab^3ab^2a^2ba$
	396	$a^2b^3ab^2a^2ba^2$
$3A$	1320	$ba^2ba^2b^2ab$
	440	ab^2abab^2abab
$3B$	2640	$a^2b^2abab^3a$
	1584	$ababa^2b^3a^2$
$3C$	7920	bab^2ab^2ab
$4A$	2970	$bababab^2aba$
$4B$	11880	a^2b
	5940	b^2
	2970	$babab^2ab$
$5A$	9504	b^2aba^2
	9504	$ba^2b^3a^2ba$
$5B$	47520	a^2b^2a
$6A$	11880	$ba^2b^2ab^2a$
	3960	$ab^2a^2baba^2ba$
$6B$	15840	a^2
	15840	a
$6C$	23760	$babab^3a$
	23760	ba^2b^2aba
	15840	bab^3aba
$6E$	23760	b^2a
	23760	a^2b^2
$7A$	95040	ba^2
	31680	bab^2aba
$8A$	23760	$babab^2a$
	23760	$b^2ababab$
$8B$	23760	$abab^2a$
	23760	b^2a^2ba
$8C$	23760	ab^2ab^2
	23760	$b^2a^2b^2a^2$
	23760	b^2aba
	23760	ba^2bab^3
$9A$	31680	bab^2ab^3
	31680	$abababab$
$9B$	63360	$ababab^2$
$10A$	47520	aba^2b^2
	47520	$babab^2$
$10B$	47520	aba
	47520	$a^2b^3a^2$
$12A$	23760	a^2ba^2
	23760	ab^3a
$12B$	47520	b
	47520	$ababa^2$
	47520	bab^2ab
$14A$	95040	$baba$
	95040	ab^2aba
$15A$	95040	ab^2
	95040	$abab$
$15B$	190080	$abab^2$
$18A$	95040	ab^2ab
	95040	$babab$
$21A$	190080	ba^2ba^2
$24A$	95040	ab^2a
	95040	$a^2b^2a^2$
$24B$	95040	a^2ba
	95040	ba
$30A$	95040	aba^2
	95040	ab^3a^2

2.16 $K \cong Co_2$

2.16.1 $X = 2A, t = a$

X_C	$\|\mathcal{O}\|$	g
$2B$	1260	bab^2ab
$2C$	1008	b^3abab^3
$3B$	14336	bab
$4C$	40320	b

2.16.2 $X = 2B, t = a(ababa)^2a(ababa)^7$

X_C	$\|\mathcal{O}\|$	g
$2A$	15	$(b^4a)^2b^3ab^2ab^4(bab)^2a(bab)^7abab^3ab^2(ab)^2$
$2B$	1680	b^3abab^3
	210	b^4ab^4abab
$2C$	2520	$(ab)^5a(ab)^2a(ab)^3$
$3B$	35840	b^2ab
$4A$	1920	bab^3ab^2abab
$4C$	20160	bab^2ab
$4E$	161280	bab
$4F$	13440	$babababab$
	13440	$b^3abab^2ab^2$
$5B$	344064	b^2
$6E$	430080	b

2.16.3 $X = 2C, t = (b^{-1}(ab^2)^2ab^{-1}(ab^2)^4)^5$

X_C	$\|\mathcal{O}\|$	g
$2A$	240	$ab^4ab^2abab^4ab^3ab^3ab^4ab^3aba$
	45	$ab^2ab^4ab^3abab^3ab^4ab^4ab^4ab^2a$
$2B$	720	b^2a
	240	$b^4ab^4ab^3ab^4ab^4ab^2ab^3ab^2a$
	180	$ab^2ab^3ab^2abab^3ab^3ab^2$
	30	$abab^4a$
$2C$	1440	$abab^3ab^3ab^2abab^2a$
	1440	$ab^3ab^4ab^3ab^2ab^2ab^4a$
	1440	$b^2ababab^3ab^4ab^3$
	1152	$abab^3ab^3ab^3ab^4ab^2a$
	360	$abababab^3ab^2abab^3ab^3ab^2$
$3A$	10240	$ab^4ab^2abab^2abab^2a$
$3B$	15360	ab^3ab^2a
	7680	b^3ab^2

. . . continued on next page

. . . continued from previous page

X_C	$\lvert\mathcal{O}\rvert$	g
	1024	$ab^2ab^2ab^3ab^3a$
$4A$	3840	$b^4ab^4ab^3ab^4ab^4ab^4$
	1440	$abab^2ab^2ab^4ab^2$
$4B$	11520	$bab^4ab^4ab^3ab^3a$
	5760	$ab^2abab^2abab^3a$
	5760	$ab^2ab^4ab^3ab^4ab^3a$
	1440	$ab^4abab^2ab^4ab^2abab^3a$
	1440	$ab^2ab^4ab^3abab^3ab^4aba$
	720	$abab^4ab^2ab^2ab^2ab^3ab^3$
	720	$b^2ab^4ab^2ab^2ab^4ab^3ab^4a$
$4C$	11520	ab^2ab^3a
	11520	$ab^3abab^3ab^2ab^2$
	5760	$b^3ab^4ab^2a$
	5760	ab^3abab^2
	2880	$abab^4ab^2a$
	1440	$b^4ab^4ab^3ab^2ab^2ab^3a$
	1440	$ab^2ab^3ab^3ab^2abab$
	720	$abababab^2abab^3a$
	720	$ab^3ab^4ab^2a$
$4D$	23040	$bab^3abab^2ab^3ab^2$
	11520	$abab^3ab^3abab^3$
	5760	$ab^4abab^4ab^2a$
	2880	$ababab^4ab^2ab^4ab^4ab^3a$
	2880	$ab^2ababab^3abab^4ab^4a$
$4E$	23040	$bababab^2$
	23040	$ab^3ab^2ab^2$
	11520	$ab^3ab^2ab^4ab^2$
	11520	$b^3abab^3ab^2a$
	5760	b^3ab^4a
	5760	$abab^2$
	5760	$b^4ab^3ab^2abab^3a$
$4F$	23040	$abab^2ab^2ab^2$
	23040	$b^3ab^3ab^3ab^4a$
	11520	$b^4ab^4ab^2ab^4ab^3ab^2a$
	11520	$ab^3ab^4ab^3ab^2ab^2ab^2$
	5760	$b^2ab^4ab^2ab^2ab^2ab^3ab^2a$
	5760	$b^2ab^2ab^3ab^3ab^4ab^2ab^3a$
	5760	$ab^3ab^2ab^4ab^2ab^3ab^3ab^2a$
	5760	$ab^4ababab^3ab^2ab^2ab^3$
	5760	$ab^2ab^3abab^2ab^2ab^3ab^3$
	5760	$b^4ab^4ab^2ab^2ab^2ab^3ab^2a$

. . . continued on next page

... continued from previous page

X_C	$\|\mathcal{O}\|$	g
	2880	$b^3ab^2abab^2$
	2880	$b^3ab^4ab^3ab^2$
	1920	$abab^4ab^2ab^2ab^4ab^3$
	1920	$bab^3ab^3abab^2ab^4$
$4G$	46080	$b^3ab^3ab^2ab^3a$
	46080	$ab^2ab^3ab^2ab^2$
	23040	$abab^2ab^2ab^3ab^2a$
	23040	$ab^3ab^2ab^3ab^3ab^4a$
$5A$	73728	$ab^2ab^3abab^2a$
$5B$	184320	b^2ab^2a
	122880	$b^2ab^3ab^2$
$6A$	92160	$bab^3ab^4ab^3ab^2a$
$6B$	122880	$b^2ab^4ab^2a$
$6C$	46080	$b^2ab^3ab^3ab^2a$
	46080	$b^3abab^2ab^2$
	15360	$b^4ab^2ab^3ab^3a$
	15360	$ab^2ab^2ab^3ab$
$6D$	92160	$b^4abab^2ab^4a$
	23040	$b^4ab^3ab^4ab^2ab^2$
	15360	ab^2ab^2aba
	15360	$ab^4ab^3ab^3a$
$6E$	92160	$babab^2$
	92160	bab^4a
	46080	$b^3ab^2ab^3ab^2a$
	46080	bab^2ab^3a
	46080	b^2abab^4a
	46080	$abab^4ab^3$
	15360	$ab^2abab^4abab^2a$
$6F$	92160	ab^3aba
	92160	ab^4ab^2a
	46080	$b^3ab^2abab^2a$
	46080	$ab^4ab^4ab^3ab^3$
	46080	$ab^3ab^4ab^3ab^2$
	46080	b^2ab^2ababa
$7A$	737280	b^4ab^2
$8A$	92160	b^2aba
	92160	ab^4ab^3
$8B$	92160	$ab^4ab^2ab^2aba$
	92160	$ab^4ab^3ab^3aba$
	46080	$abab^3ab^2a$

... continued on next page

. . . continued from previous page

X_C	$\lvert\mathcal{O}\rvert$	g
	46080	$ab^3ab^2ab^4a$
	23040	$bab^3ab^3abab^3$
	23040	$b^2ab^4ab^2ab^2ab^4$
$8C$	92160	$b^3ab^3ab^3ab^4$
	92160	b^2
	92160	b^3
	92160	$bab^2ab^2ab^2$
$8D$	92160	$b^3ab^3ab^2$
	92160	$ab^2ab^3ab^3ab^4ab^2$
	92160	$b^4ab^3ab^3a$
	92160	bab^2abab^3a
	92160	$b^3ab^2ab^2$
$8E$	92160	ab^3a
	92160	ab^2a
	46080	$ab^3abab^4ab^2$
	46080	$b^3abab^4ab^2a$
	46080	$ab^2ab^2abab^3$
	46080	b^2ababa
	46080	$b^2ab^4ab^3ab^3a$
	46080	$b^2ab^2ab^3ab^3a$
	46080	$ab^2ab^2ab^3ab^3$
	46080	$ab^4ab^4ab^3$
	23040	$ab^4ab^2ab^2abab^3$
	23040	$b^4ab^4ab^2ab^4ab^3$
$8F$	184320	ab^4abab^3a
	184320	ab^2ab^4aba
	92160	$abab^3abab^2ab^2$
	92160	$b^3ab^4ab^4ab^3ab^2a$
$9A$	737280	ab^4abab^2
	245760	$b^2ab^4ab^2$
$10A$	368640	$b^2ab^3ab^2a$
$10B$	368640	ab^2ab^2
	368640	b^3ab^3a
$10C$	368640	ab^4
	368640	$abab^3abab^3a$
$11A$	1474560	b^2ab^3a
$12A$	122880	bab^2abab^4a
	122880	$b^4abab^4ab^3$
$12B$	368640	$b^2abab^3ab^2$
	184320	$ab^4ab^2ab^4ab^2$

. . . continued on next page

... continued from previous page

X_C	$\lvert\mathcal{O}\rvert$	g
$12C$	368640	$ab^2ab^2ab^3ab^2$
	184320	$ab^4abab^2ab^2$
$12D$	184320	$b^4abab^2ab^2$
	184320	$ababab^3ab^3a$
	92160	$ab^4ab^2ab^3a$
	92160	ab^2ab^3aba
$12E$	368640	$b^3ab^2ab^4$
	368640	bab^3ab^2
$12F$	184320	b^2abab^2a
	184320	ab^3ab^4a
	184320	$abab^3ab^3$
	184320	$abab^2a$
	92160	ab^2aba
	92160	ab^4ab^3a
$12G$	368640	bab^4ab^2a
$12H$	184320	$b^4ab^3ab^3ab^2$
	184320	$ab^3ab^2ab^3a$
	184320	$ababab^4ab^2a$
	184320	$ab^4ab^3ab^4ab^2a$
	184320	$b^3ab^2ab^2ab$
	184320	$ab^2ab^3ab^2a$
$14A$	737280	ab^4a
$15A$	737280	b^4a
	737280	bab^2a
$16A$	368640	ab^3ab^2
	368640	ab^2abab^4
	368640	b^3ab^2a
	368640	bab^4ab^3a
$18A$	737280	b^3ab^3
$20A$	737280	ab^4ab^2
	737280	b^3aba
$20B$	737280	ab^4ab^4a
	737280	$ababa$
$24A$	737280	ab^2
	737280	b^3a
$24B$	737280	b^4
	737280	b
$28A$	737280	$abab^3ab^2$
	737280	$ab^4ab^2ab^2$
$30A$	737280	b^2ab^4

... continued on next page

... continued from previous page

X_C	$\|\mathcal{O}\|$	g
	737280	bab^3

References

[1] Bates, C. and Rowley, P. Involutions in Conway's largest simple group. LMS J. Comput. Math. 7 (2004), 337–351 (electronic).

[2] Bing, R.H. A homeomorphism between the 3-sphere and the sum of two solid horned spheres. Ann. of Math. (2) 56, (1952). 354–362.

[3] Bosma, W., Cannon, J. and Playoust, C. The Magma algebra system. I. The user language, J. Symbolic Comput., 24 (1997), 235265.

[4] Brauer, R. and Fowler, K.A. On groups of even order. Ann. of Math. (2) 62 (1955), 565–583.

[5] Conway, J.H., Curtis, R.T., Norton, S.P., Parker, R.A. and Wilson, R.A. Atlas of Finite Groups, Clarendon, Oxford, (1985).

[6] Edwards, C.M. and Rüttimann, G.T. Involutive and Peirce gradings in JBW*-triples. Comm. Algebra 31 (2003), no. 6, 2819–2848.

[7] Feit, W. and Thompson, J.G. Solvability of groups of odd order. Pacific J. Math. 13 1963 775–1029.

[8] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.3*; 2002, `http://www.gap-system.org`.

[9] Gorenstein, D. Finite groups. Second edition. Chelsea Publishing Co., New York, 1980.

[10] Gorenstein, D. The classification of finite simple groups. Vol. 1. Groups of noncharacteristic 2 type. The University Series in Mathematics. Plenum Press, New York, 1983. x+487 pp.

[11] Gorenstein, D., Lyons, R. and Solomon, R. The classification of the finite simple groups. Mathematical Surveys and Monographs, 40.1. American Mathematical Society, Providence, RI, 1994. xiv+165 pp.

[12] Górnicki, J. and Rhoades, B. E. A general fixed point theorem for involutions. Indian J. Pure Appl. Math. 27 (1996), no. 1, 13–23.

[13] Montgomery, D. and Zippin, L. Examples of transformation groups. Proc. Amer. Math. Soc. 5, (1954). 460–465.

[14] Montgomery, S. Fixed rings of finite automorphism groups of associative rings. Lecture Notes in Mathematics, 818. Springer, Berlin, 1980. vii+126 pp.

[15] The Smith conjecture. Papers presented at the symposium held at Columbia University, New York, 1979. Edited by John W. Morgan and Hyman Bass. Pure and Applied Mathematics, 112. Academic Press, Inc., Orlando, FL, 1984. xv+243 pp.

[16] Springer, T. A. The classification of involutions of simple algebraic groups. J. Fac. Sci. Univ. Tokyo Sect. IA Math. 34 (1987), no. 3, 655–670.

[17] Springer, T. A. Some results on algebraic groups with involutions. Algebraic groups and related topics (Kyoto/Nagoya, 1983), 525–543, Adv. Stud. Pure Math., 6, North-Holland, Amsterdam, 1985.

[18] Rowley, P. and Taylor, P. Point-line collinearity graphs of two sporadic minimal parabolic geometries. J. Algebra 331 (2011), 304-310.

[19] Rowley, P. and Taylor, P. Involutions in Janko's simple group J_4. To appear, LMS J. Comput. Math.

[20] Taylor, P. Involutions in Fischer's sporadic groups. Preprint `http://eprints.ma.man.ac.uk/1622/`

[21] Wilson, R. Standard generators for sporadic simple groups. J. Algebra 184 (1996), no. 2, 505–515.

[22] Wilson, R., Walsh, P., Tripp, J., Suleiman, I., Rogers, S., Parker, R., Norton, S., Linton, S. and Bray, J. ATLAS of finite group representations, `http://web.mat.bham.ac.uk/atlas/v2.0/`.