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Involutions in Fischer's sporadic groups

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Abstract

For each of Fischer's sporadic simple groups and their automorphism groups, we catalogue the suborbits of the conjugation action on each of the involution conjugacy classes. Representative elements for the suborbits are provided, where possible as words in the standard generators and elsewhere they are given electronically as base images. Additionally we use this data to determine the structure of the commuting involution graphs on the two largest involution classes in Fi_{24}.

1 Introduction

The three sporadic groups Fi_{22}, Fi_{23} and $Fi_{24}{}'$ arose from Fischer's classification of the 3-transposition groups. The defining property of such a group is that it is generated by a conjugacy class of involutions (usually referred to as the 'transpositions' by analogy with the symmetric groups), products of any two of which have order not exceeding three. Fischer's classification of these groups gave rise to three exceptional examples, now named Fi_{22}, Fi_{23} and Fi_{24}, the first two of which were also new sporadic simple groups, as was the derived subgroup $Fi_{24}{}'$ of the third. The groups were constructed by considering the automorphism group of the graph which has the class of transpositions as its vertices; a pair of vertices joined by an edge if and only if they commute. The involutions in these classes are therefore obviously well understood. But the groups have other involution classes besides the transpositions, so we might wish to investigate these as well.

An obvious question is then: what is the structure of the equivalent graphs on the other classes of involutions? This type of graph is known as a 'commuting involution graph' and has in recent years been studied for a wide variety of groups (see, for example, [1], [2], [6]). In particular, in [3] the graph structure was determined for all involution classes in the Fischer groups save two largest classes of Fi_{24}. We determine the structure of these outstanding cases as part of the present work: that is, we discover the diameters of the graphs as well as the sizes of the disks of vertices at a given distance from a fixed vertex.

Study of the commuting involution graph leads naturally to consideration of the suborbit structure of the class, that is the orbits under the action of a point stabilizer: $C_G(t)$ for t an element from the relevant class of involutions. This is because, once an involution $t \in X$ is fixed, the set of vertices at a given distance from it in the graph is a union of these suborbits. It is the cataloguing of the $C_G(t)$-orbits that is the main focus of the present work.

Throughout this paper, G denotes one of the groups Fi_{22}, Fi_{23}, ${Fi_{24}}'$ or one of their automorphism groups $Fi_{22}:2$ and Fi_{24} (Fi_{23} having no outer automorphisms). Also, X is a G-conjugacy class of involutions. In the online ATLAS [11], generators for these groups are provided in GAP [7] and MAGMA [4] formats. We employ the latter package for our computation. ATLAS [5] conventions are adopted for the names of conjugacy classes.

For each pair (G, X) under consideration we fix an element $t \in X$, and then we wish to determine the size of, and find a representative element $x \in X$ for, each $C_G(t)$-orbit on X. For all but the pairs $({Fi_{24}}', 2B)$ and $(Fi_{24}, 2D)$ we give these representatives in terms of words in the generators a, b (or c, d for an automorphism group), these words giving us elements g that conjugate t to the desired representatives. The online ATLAS uses standard generators [10] so these words can be used to derive explicit representatives in any representation of the group.

In the cases $({Fi_{24}}', 2B)$, $(Fi_{24}, 2D)$ we give representatives as base images for a specified base and strong generating set, using the smallest-degree permutation representations of these groups available in [11]. We also in these two cases uncover the structure of the commuting involution graph $\mathcal{C}(G, X)$.

In the following section we summarise the main results. Section 3 describes how these were obtained, and Section 4 tabulates the full data on the $C_G(t)$-orbits.

This work also appears in the author's PhD thesis [9].

2 Results

For each of the involution conjugacy classes in the Fischer sporadic groups and their automorphism groups, Table 1 gives the number of suborbits and the number of sets X_C into which they fall—these sets are defined in the following section. We also outline the structure of the commuting involution graphs for $({Fi_{24}}', 2B)$ and $(Fi_{24}, 2D)$. In Section 4 we provide tables listing the full details of the $C_G(t)$-orbits for all thirteen pairs (G, X) except for the classes of transpositions: $2A$ in Fi_{22} and Fi_{23} and class $2C$ in Fi_{24}.

G	X	Suborbits	Nonempty X_C
Fi_{22}	$2A$	3	3
	$2B$	14	11
	$2C$	136	57
$Fi_{22}:2$	$2D$	4	4
	$2E$	56	31
	$2F$	74	38
Fi_{23}	$2A$	3	3
	$2B$	12	10
	$2C$	303	92
Fi_{24}'	$2A$	13	11
	$2B$	233	104
Fi_{24}	$2C$	3	3
	$2D$	232	84

Table 1: Involution suborbits in the Fischer groups

2.1 Commuting Involution Graphs

Knowledge of the $C_G(t)$-orbits allows us to easily compute the structure of the commuting involution graphs for the two previously unknown cases, which we summarise here in terms of their disk structures. For our fixed $t \in X$ and $i \in \mathbb{N}$, the ith disk is defined as $\Delta_i(t) = \{x \in X \mid d(t,x) = i\}$ where $d(-,-)$ is the usual distance metric on $\mathcal{C}(G,X)$. A simple argument demonstrates that the graph is vertex-transitive so that the disk sizes are the same whatever $t \in X$ we choose, and that once t is fixed each of the discs $\Delta_i(t)$ is a disjoint union of $C_G(t)$-orbits.

Theorem 1. *Let $\mathcal{C}(G,X)$ be the commuting involution graph of $G \cong Fi_{24}'$ on $X = 2B$. Then for $t \in X$,*

- *$\Delta_0(t) = \{t\}$ has size 1 and is composed of one $C_G(t)$-orbit.*
- *$\Delta_1(t)$ has size 3,324,762 and is composed of six $C_G(t)$-orbits.*
- *$\Delta_2(t)$ has size 3,755,093,739,776 and is composed of one hundred and seventy-four $C_G(t)$-orbits.*
- *$\Delta_3(t)$ has size 4,064,208,224,256 and is composed of fifty-two $C_G(t)$-orbits.*

The $C_G(t)$-orbits comprising the graph are as detailed in Table 10.

Theorem 2. *Let $\mathcal{C}(G,X)$ be the commuting involution graph of $G \cong Fi_{24}$ on $X = 2D$. Then for $t \in X$,*

- *$\Delta_0(t) = \{t\}$ has size 1 and is composed of one $C_G(t)$-orbit.*

- *$\Delta_1(t)$ has size 3,682,503 and is composed of six $C_G(t)$-orbits.*
- *$\Delta_2(t)$ has size 822,139,288,316 and is composed of one hundred and ninety-one $C_G(t)$-orbits.*
- *$\Delta_3(t)$ has size 2,021,240,770,560 and is composed of thirty-six $C_G(t)$-orbits.*

The $C_G(t)$-orbits comprising the graph are as detailed in Table 11.

3 Computation

All computation is carried out in the smallest-degree permutation representation of the relevant group available in [11]. In these representations the necessary computations (testing for $C_G(t)$-conjugacy and computing the order of $C_{C_G(t)}(x)$ for a representative $x \in X$ to find the orbit size) are straightforward.

Our initial strategy is to build up a list of $C_G(t)$-orbit representatives by random search. We take a random element $g \in G$ and form $x = t^g \in X$. This element is tested for $C_G(t)$-conjugacy against all elements in the list thus far. If it is found to be a representative for a new orbit, its size is calculated.

However, given the size of the groups at hand and the number of suborbits to be located, this strategy would take a long time to find every suborbit. The following tricks are employed to speed up the search.

3.1 Structure Constants

Definition 3. *For G, X and t as above, and C a further G-conjugacy class, we define*

$$X_C = \{x \in X \mid tx \in C\}.$$

It is easy to see that for any class C of G, the set X_C is a (possibly empty) union of $C_G(t)$-orbits on X. Further, the values of $|X_C|$ are readily calculated from the character table (for instance using the GAP function `ClassMultiplicationCoefficient`). So we can partition X into the subsets X_C and consider each one in turn, knowing from their sizes when we have found all the $C_G(t)$ orbits that comprise such a set.

In practice it is often not a simple matter to quickly determine the class of a given $z = tx$, so this information cannot be used directly. However it is obviously simple to determine the order of z so we can at least check when we have found all the suborbits with a given order of z. Then if a random $x \in X$ is formed later having z of that order, we can discard it without having to carry out any $C_G(t)$-conjugacy testing.

Once all the suborbits have been located, we determine the class of $z = tx$ for every representative x. In Section 4, the suborbits are listed grouped according to which set X_C they are contained in.

3.2 Powering Suborbits

We note the following observation.

Lemma 4. *Let $x \in X_C$, so that $z = tx \in C$ and suppose n is a divisor of the order of z. Let D be the conjugacy class containing z^n. We define the element $x^{(n)} = tz^n$, and note the following properties.*

(i) $x^{(n)} \in X_D$;

(ii) if $x, y \in X$ are $C_G(t)$-conjugate then so are $x^{(n)}$ and $y^{(n)}$; and

(iii) All $C_G(t)$-conjugates of $x^{(n)}$ arise as $y^{(n)}$ for some $C_G(t)$-conjugate y of x.

Proof. We see that $x^{(n)} = t(tx)^n = xtx \dots tx$, so clearly $x^{(n)} \in X$, and since $tx^{(n)} = z^n \in D$ we obtain (i). From the definition it is immediate that if $h \in C_G(t)$ conjugates x to y then it also conjugates $x^{(n)}$ to $y^{(n)}$, giving (ii). Similarly if $(x^{(n)})^h = y$ for $h \in C_G(t)$ then $(x^h)^{(n)} = y$ and so we get (iii). $\square$

When we have found a representative x for a new suborbit, we form and check $x^{(n)}$ for each n dividing the order of z. This allows us much more easily to alight on suborbits which are small and so hard to find by random searching, but which can be 'powered down' to in this manner from much larger orbits. For example, when $G \cong Fi_{22}$ and X is the class $2C$, there is a suborbit in X_{2C} of size 9, which we would be very unlikely to find in a random search, however, a representative is easily found as $x^{(4)}$ for x in an orbit of size 55296 contained in X_{8A}.

3.3 Suborbit invariants

While $C_G(t)$-conjugacy testing in the groups we consider is possible, it is relatively computationally expensive. So to minimize its use we compute for each $x \in X$ considered two suborbit invariants: the cycle type of $z = tx$ and the orbit sizes of $\langle t, x \rangle$. Elements having different invariants are not $C_G(t)$-conjugate so we avoid some extra computation.

3.4 Inverse suborbits

Our suborbit representatives are formed by taking random elements $g \in G$ and setting $x = t^g$. Now consider the element $x' = t^{g^{-1}}$. We note the following properties:

Lemma 5. *Let $t \in X$, $g \in G$. Set $x = t^g$ and $x' = t^{g^{-1}}$. Then*

(i) the $C_G(t)$-orbits containing x and x' are of the same size;

(ii) x and x' lie in the same X_C; and

(iii) if $y = t^h$ is $C_G(t)$-conjugate to x then $y' = t^{h^{-1}}$ is $C_G(t)$-conjugate to x'.

Proof. We have that $C_{C_G(t)}(x) = C_G(t) \cap C_G(t^g)$ and $C_{C_G(t)}(x') = C_G(t) \cap C_G(t^{g^{-1}})$, so clearly $C_{C_G(t)}(x)^{g^{-1}} = C_{C_G(t)}(x')$. Then $|C_{C_G(t)}(x)| = |C_{C_G(t)}(x')|$ and so their orbits have the same size, giving (i). For (ii), we note that $(tx)^{g^{-1}t} = tx'$. Now let $c \in C_G(t)$ so that $x^c = y$. So $t^{gc} = t^h$, that is, $gch^{-1} \in C_G(t)$. Then we see that $x'^{(gch^{-1})} = t^{g^{-1}(gch^{-1})} = t^{ch^{-1}} = t^{h^{-1}} = y'$, giving (iii). □

So when we find a representative x of a new $C_G(t)$-orbit, we can form the element x', and we need only to check it for $C_G(t)$-conjugacy against x to see if it represents a new suborbit (if it were in a previously discovered suborbit then when that orbit were found we would have uncovered the orbit $x^{C_G(t)}$ by the same process). In some of the classes we consider a large number of the $C_G(t)$-orbits fall into pairs of 'inverse' orbits in this manner so use of this technique can save much computational time.

4 Suborbit tables

In each of Tables 2–9, each row corresponds to a $C_G(t)$-orbit (note that a row may extend to more than one line if there is a long entry in the third column; each orbit size listed in the second column marks the start of a new row). The first column gives the class C of G for which the suborbit lies in X_C. The second column gives the size of the suborbit and the third gives information on how to obtain a representative element for that suborbit. This is either a word in the standard generators of G giving an element g so that t^g is a suborbit representative; or a list of symbols C_i' or D_i for D a class of G: the former meaning that the suborbit is the inverse of the i^{th} listed suborbit in X_C and the latter meaning the suborbit can be powered to from the ith listed suborbit in X_D.

For Tables 10 and 11, covering the two larger examples $(Fi_{24}', 2B)$ and $(Fi_{24}, 2D)$, words for representative elements are not provided. Instead, these tables simply list the sizes of the $C_G(t)$-orbits, and which disk of the commuting involution graph $\mathcal{C}(G, X)$ each suborbit lies in. Where two or more suborbits in a particular X_C have the same size and lie in the same disk, their entries are collapsed into one row of the table. This is denoted by an entry of the form $C_{1,2}$ in the first column. Representative elements for these groups may be downloaded from `http://www.maths.manchester.ac.uk/~ptaylor/Fi24/`,

where they are stored as base images (relative to a base which is also provided). We also provide there a partial list of $C_G(t)$-orbit representatives for the class $2D$ in the group $G \cong 3.Fi_{24}$. This class is of interest because G occurs as a maximal subgroup in the Monster sporadic group $\mathbb{M}$, hence these suborbits can be used to investigate the so-called 'Monster graph' [8].

Table 2: $G \cong Fi_{22}$, $X = 2B$, $t = (ababab^3)^{12}$.

C	Orbit size	Representative
$1A$	1	-
$2B$	270	$4A_1$
	360	$4E_1$ $6D_1$
	1152	$4E_2$ $6I_1$
$2C$	4320	$4D_1$
$3A$	1024	$6D_1$
$3C$	40960	$6I_1$
$4A$	34560	$babab^3a$
$4D$	138240	b^2a
$4E$	69120	aba
	69120	$4E'_1$
$5A$	442368	ba
$6D$	46080	b^2
$6I$	368640	b

Table 3: $G \cong Fi_{22}$, $X = 2C$, $t = ((ababab^3)^2ab^3abab^3)^6$.

C	Orbit size	Representative
$1A$	1	t
$2A$	48	$6A_1$ $6A_2$ $6B_1$ $6E_1$ $6E_2$ $10A_1$ $10A_2$ $14A_1$ $14A_2$
$2B$	9	$4A_1$ $4C_2$ $4E_1$ $4E_4$
	216	$4A_3$ $4B_1$ $4C_1$ $4E_5$ $6D_1$ $6D_2$
	288	$4B_3$ $4E_7$ $4E_10$ $6D_4$ $6I_1$ $10B_1$
	576	$4A_2$ $4B_2$ $4B_4$ $4C_3$ $4E_2$ $4E_3$ $4E_6$ $4E_8$ $4E_9$ $6C_1$ $6D_3$ $6D_5$ $6I_2$ $6I_3$ $10B_2$
$2C$	432	$2C'_2$ $6F_1$
	432	$2C'_1$ $6F_2$
	432	$b^5ab^4abababababab^4$
	432	$4D_1$ $4D_2$ $4D_3$
	3456	$4D_4$ $6F_3$ $6F_4$ $6G_1$ $6H_1$ $6J_1$ $6J_2$ $6K_1$
$3A$	768	$6A_1$ $6D_2$ $6D_3$ $6D_4$ $6F_2$ $6F_4$ $15A_1$
	1536	$6A_2$ $6D_1$ $6D_5$ $6F_1$ $6F_3$ $15A_2$ $21A_1$
$3B$	8192	$6B_1$ $6C_1$ $6G_1$ $9A_1$ $9B_1$ $9B_2$
$3C$	12288	$6E_2$ $6I_3$ $6J_2$
	24576	$6E_1$ $6H_1$ $6I_1$ $6I_2$ $6J_1$
$3D$	98304	$6K_1$ $9C_1$
$4A$	1152	$8A_2$ $8B_1$
	9216	$8A_1$ $8A_3$ $8B_2$ $8B_4$ $12A_1$ $12A_2$ $12C_5$
	13824	$8A_4$ $8B_3$ $12C_1$ $12C_2$ $12C_3$ $12C_4$
$4B$	6912	$4B'_2$ $12D_2$
	6912	$4B'_1$ $12D_1$
	13824	$4B'_4$ $12D_3$ $12J_1$ $20A_2$
	13824	$4B'_3$ $12D_4$ $12J_2$ $20A_1$
$4C$	6912	$ababab^4ababab^2$
	6912	$4C'_1$
	27648	$8C_1$ $12H_1$ $12H_2$

$4D$	13824	$8D_3$
	13824	$8D_2$
	13824	$8D_5$
	110592	$8D_1$ $8D_4$ $8D_6$ $12E_1$ $12E_2$ $12F_1$ $12F_2$
		$12G_1$ $12G_2$ $12K_1$ $12K_2$
$4E$	1152	b^2ab^6abab
	1152	$4E_1'$
	6912	$4E_4'$
	6912	$b^3ab^2ab^4ab^2a$
	13824	$4E_6'$ $12I_2$
	13824	$4E_5'$ $12I_1$
	13824	$abab^6abab^2$
	13824	$4E_7'$
	27648	$4E_10'$ $12I_4$
	27648	$4E_9'$ $12I_3$
$5A$	147456	$10A_2$ $10B_1$ $15A_1$
	221184	$10A_1$ $10B_2$ $15A_2$
$6A$	9216	$6A_2'$ $30A_1$
	9216	$6A_1'$ $30A_2$
$6B$	24576	$18C_1$ $18C_2$
$6C$	73728	$12A_1$ $12A_2$ $12H_1$ $12H_2$ $18D_1$
$6D$	13824	$12C_1$ $12C_3$ $12D_2$ $12I_2$
	13824	$12C_2$ $12C_4$
	18432	$12D_1$ $12I_4$
	18432	$12D_3$ $12I_3$
	55296	$12C_5$ $12D_4$ $12I_1$
$6E$	73728	$6E_2'$
	73728	bab^6
$6F$	13824	b^5aba
	13824	$6F_1'$
	55296	b^5ab
	55296	$6F_3'$
$6G$	221184	$12E_1$ $12E_2$ $12F_1$ $12F_2$
$6H$	221184	$12G_1$ $12G_2$
$6I$	73728	$12J_1$
	73728	ab^2ab^3a
	110592	$12J_2$
$6J$	221184	$6J_2'$
	221184	$ababab^4a$
$6K$	884736	$12K_1$ $12K_2$
$7A$	294912	$14A_1$
	884736	$14A_2$ $21A_1$
$8A$	55296	b^2ab^6
	55296	$8A_1'$
	110592	$8A_4'$ $24B_1$
	110592	$8A_3'$ $24B_2$
$8B$	18432	ab^2ab^7
	18432	$8B_1'$
	110592	$8B_4'$ $24A_2$

	110592	$8B_3'$ $24A_1$
$8C$	221184	$ab^2ab^3ab^2$
$8D$	221184	$8D_5'$
	221184	ab^3ab^2
	221184	$8D_6'$
	221184	$8D_2'$
	221184	$abab^2a$
	221184	b^3ab^6a
$9A$	294912	$18D_1$
$9B$	294912	$18C_2$
	884736	$18C_1'$
$9C$	1769472	$baba$
$10A$	442368	$10A_2'$ $30A_2$
	442368	$10A_1'$ $30A_1$
$10B$	442368	$20A_2$
	442368	$20A_1$
$12A$	73728	ab^3ab^2ab
	73728	$12A_1'$
$12C$	110592	ab^5aba
	110592	$12C_1'$
	110592	$24A_2$
	110592	$24B_2$
	221184	$24A_1$ $24B_1$
$12D$	110592	$12D_2'$
	110592	ab^3a
	221184	$12D_4'$
	221184	b^2abab^2a
$12E$	442368	$12E_2'$
	442368	b
$12F$	442368	$12F_2'$
	442368	ba
$12G$	442368	bab^5
	442368	$12G_1'$
$12H$	221184	ab^4a
	221184	bab^2abab
$12I$	110592	$12I_2'$
	110592	b^5abab^2a
	221184	$12I_4'$
	221184	b^2abab^4
$12J$	442368	$12J_2'$
	442368	bab^3ab
$12K$	884736	ab
	884736	$12K_1'$
$13A$	1769472	b^5a
$13B$	1769472	ab^3
$14A$	884736	$14A_2'$
	884736	b^4a
$15A$	884736	$30A_1$
	884736	$30A_2$

$18C$	884736	$abab^3a$
	884736	$18C_1'$
$18D$	884736	b^4
$20A$	884736	bab
	884736	$20A_1'$
$21A$	1769472	aba
$24A$	442368	$24A_2'$
	442368	b^6
$24B$	442368	$24B_2'$
	442368	$abab$
$30A$	884736	bab^2
	884736	$30A_1'$

Table 4: $G \cong Fi_{22} : 2$, $X = 2D$, $t = d^9$.

C	Orbit size	Representative
$1A$	1	t
$2B$	1575	$4A_1$
$3C$	22400	$cdcdc$
$4A$	37800	cd^4c

Table 5: $G \cong Fi_{22} : 2$, $X = 2E$, $t = (d^6cdc)^9$.

C	Orbit size	Representative
$1A$	1	t
$2B$	27	$4A_1$ $4A_2$ $4E_2$
	540	$4C_1$ $4E_4$ $6D_2$ $6I_3$ $10B_3$
	1080	$4A_3$ $4C_2$ $4E_1$ $4E_3$ $6C_1$ $6D_1$ $6D_3$ $6I_1$ $6I_2$ $6I_4$ $10B_1$ $10B_2$
$2C$	3240	$4D_1$ $6H_1$
$3A$	2304	$6D_1$ $6D_2$ $6D_3$ $15A_1$
$3B$	5120	$6C_1$ $9A_1$ $9B_1$
$3C$	5760	$6I_2$
	11520	$6H_1$ $6I_1$ $6I_3$ $6I_4$
$4A$	2160	$8B_2$ $8B_4$
	3240	$8A_1$
	17280	$8A_2$ $8B_1$ $8B_3$ $12A_1$ $12A_2$ $12B_1$
$4C$	25920	$8C_1$
	51840	$8C_2$ $12G_1$
$4D$	103680	$12F_1$
$4E$	8640	$4E_2'$
	8640	d^2cdcd^5c
	51840	$4E_4'$ $12H_1$
	51840	$4E_3'$ $12H_2$
$5A$	27648	$10B_1$
	414720	$10B_2$ $10B_3$ $15A_1$
$6C$	138240	$12A_1$ $12A_2$ $12G_1$ $18C_1$
$6D$	34560	$12H_1$
	34560	$12H_2$
	103680	$12B_1$

$6H$	103680	$12F_1$
$6I$	103680	$6I_2'$
	103680	cd
	138240	cd^8c
	138240	$cdcd$
$7A$	552960	d^2cd^3
$8A$	103680	cd^2cdcd^2
	103680	$8A_1'$
$8B$	34560	$d^2cd^4cdcd^3$
	34560	$8B_1'$
	103680	$8B_4'$
	103680	d^5cd^2
$8C$	414720	$16A_2$
	414720	$16A_1$
$9A$	552960	$18C_1$
$9B$	552960	cd^3c
$10B$	414720	$10B_2'$
	414720	cd^2cd
	829440	$dcdcdcd^3c$
$11A$	3317760	dcd^2cd
$12A$	138240	d^2c
	138240	$12A_1'$
$12B$	414720	cd^3
$12F$	829440	d^2cd^2cdc
$12G$	829440	d^2
$12H$	414720	$dcdcd$
	414720	$12H_1'$
$15A$	1658880	d^3
$16A$	1658880	d^2cd
	1658880	$16A_1'$
$18C$	1658880	dcd

Table 6: $G \cong Fi_{22} : 2$, $X = 2F$, $t = (d^3c)^{15}$.

C	Orbit size	Representative
$1A$	1	t
$2B$	63	$4E_1$ $6I_1$
	315	$4A_1$ $4A_2$ $4E_2$ $4E_4$ $6D_1$ $6D_2$ $6D_5$
	945	$4A_3$ $4C_1$ $4E_3$ $6C_1$ $6D_3$ $6D_4$ $6D_6$ $6I_2$ $6I_3$ $6I_4$ $10B_1$
$2C$	3780	$4D_1$ $6F_1$ $6F_2$ $6H_1$ $6H_2$ $6K_1$
$3A$	56	$6D_1$ $6D_4$
	240	$6D_2$ $6F_1$
	2520	$6D_3$ $6D_5$ $6D_6$ $6F_2$ $15A_1$ $21A_1$
$3B$	4480	$6C_1$ $9A_1$ $9A_2$ $9B_1$
$3C$	2240	$6H_1$ $6I_1$ $6I_3$
	20160	$6H_2$ $6I_2$ $6I_4$
$3D$	53760	$6K_1$ $9C_1$
$4A$	3780	$8B_2$ $12B_1$ $12C_1$ $12C_2$
	11340	$8A_2$ $12B_2$ $12C_3$ $12C_4$

	15120	$8A_1\ 8B_1\ 12A_1\ 12A_2\ 12B_3$
$4C$	45360	$8C_1\ 12G_1$
$4D$	60480	$12J_1\ 12J_2$
$4E$	15120	$d^2cd^3cd^3cdcd^3cd$
	15120	$4E'_1$
	45360	$4E'_4\ 12H_2\ 12H_3$
	45360	$4E'_3\ 12H_1\ 12H_4$
$5A$	241920	$10B_1\ 15A_1$
$6C$	120960	$12A_1\ 12A_2\ 12G_1\ 18C_1\ 18C_2$
$6D$	2520	$12C_1\ 12H_4$
	15120	$12B_1\ 12C_2\ 12C_3$
	15120	$d^2cd^4cdcd^3cd^2$
	15120	$6D'_3$
	22680	$12B_2\ 12C_4\ 12H_1$
	90720	$12B_3\ 12H_2\ 12H_3$
$6F$	60480	$6F'_2$
	60480	$d^2cd^4cd^2cd^2cd$
$6H$	60480	$6H'_2$
	60480	d^2cd^4cdcd
$6I$	20160	dcd^2cd^2cd
	60480	dcd^2cdcd^2cdcd
	60480	$6I'_2$
	181440	d^2cd^6cd
$6K$	483840	$12J_1\ 12J_2$
$7A$	1451520	$21A_1$
$8A$	181440	$8A'_2\ 24B_2$
	181440	$8A'_1\ 24B_1$
$8B$	181440	$8B'_2\ 24A_2$
	181440	$8B'_1\ 24A_1$
$8C$	362880	d^2cdcd^4cd
$9A$	80640	$18C_2$
	725760	$18C_1$
$9B$	161280	$dcdcdcd^2cdcdcd$
$9C$	967680	$dcdcdcdcdcd$
$10B$	725760	dcd
$11A$	2903040	d^2cdcd^2
$12A$	120960	dcd^5cd^4cd
	120960	$12A'_1$
$12B$	60480	$24A_1$
	181440	$24B_1$
	362880	$24A_2\ 24B_2$
$12C$	30240	dcd^7cd^3cdcd
	30240	$12C'_1$
	90720	$dcdcd^4cdcdcd$
	90720	$12C'_3$
$12G$	725760	$d^2cd^5cd^2cd$
$12H$	181440	$d^2cdcd^2cd^3cd$
	181440	$12H'_1$
	181440	$12H'_4$

	181440	dcd^2cdcd
$12J$	483840	dcd^2cd^2cdcd
	483840	$12J_1'$
$15A$	1451520	d^2cd^5cd
$18C$	725760	$18C_2'$
	725760	dcd^4cd^2cd
$21A$	2903040	dcd^3cd
$24A$	725760	$24A_2'$
	725760	dcd^3cdcd
$24B$	725760	$24B_2'$
	725760	d^2cdcd

Table 7: $G \cong Fi_{23}$, $X = 2B$, $t = a$.

C	Orbit size	Representative
$1A$	1	t
$2B$	1386	$4C_1$ $6C_1$
	12672	$4C_2$ $6K_1$
$2C$	62370	$4B_1$
$3A$	5632	$6C_1$
$3C$	630784	$6K_1$
$4B$	7983360	$babab$
$4C$	1596672	$babab^2abababab$
	1596672	$4C_1'$
$5A$	14598144	b
$6C$	709632	$b^2ab^2abab^2ababab^2abababab$
$6K$	28385280	bab

Table 8: $G \cong Fi_{23}$, $X = 2C$, $t = ((b^2a)^3ba)^6$.

C	Orbit size	Representative
$1A$	1	-
$2A$	180	$6A_1$ $6A_2$ $6A_3$ $6B_1$ $6E_1$ $6E_2$ $6E_3$ $6E_4$ $6J_1$ $6J_2$ $10A_1$ $10A_2$ $10A_3$ $10A_4$ $14A_1$ $14A_2$ $14A_3$ $14A_4$ $26A_1$ $26A_2$ $26B_1$ $26B_2$
$2B$	540	$4A_2$ $4C_1$ $4C_3$ $4C_5$ $4C_10$ $6C_2$ $6K_2$ $10B_2$
	3456	$4A_1$ $4A_3$ $4C_2$ $4C_4$ $4C_6$ $4C_7$ $4C_8$ $4C_12$ $6C_1$ $6C_6$ $6D_2$ $6K_1$ $6K_3$ $6K_4$ $10B_1$ $10B_6$ $14B_3$ $22A_1$
	4320	$4A_4$ $4C_9$ $4C_11$ $6C_3$ $6C_4$ $6C_5$ $6D_1$ $6K_5$ $6K_6$ $6K_7$ $10B_3$ $10B_4$ $10B_5$ $14B_1$ $14B_2$ $14B_4$
$2C$	810	$4B_1$ $4B_2$ $4B_3$ $4D_2$
	12960	$2C_3'$ $6I_2$ $6M_1$ $10C_2$
	12960	$2C_2'$ $6I_3$ $6M_2$ $10C_1$
	12960	$4B_4$ $4B_5$ $4D_1$ $6I_1$ $6I_4$
	103680	$4B_6$ $4B_7$ $4D_3$ $6F_1$ $6G_1$ $6G_2$ $6H_1$ $6I_5$ $6I_6$ $6I_7$ $6L_1$ $6L_2$ $6L_3$ $6L_4$ $6M_3$ $6M_4$ $6M_5$ $6M_6$ $6N_1$ $6N_2$ $6O_1$ $6O_2$ $10C_3$ $10C_4$
$3A$	1536	$6A_1$ $6C_1$ $6C_2$ $6C_3$ $6I_1$ $6I_3$ $6I_6$ $15A_1$

	23040	$6A_2$ $6A_3$ $6C_4$ $6C_5$ $6C_6$ $6I_2$ $6I_4$ $6I_5$ $6I_7$ $15A_2$ $15A_3$ $21A_1$ $21A_2$ $39A_1$ $39B_1$
$3B$	81920	$6B_1$ $6D_1$ $6D_2$ $6F_1$ $6H_1$ $9A_1$ $9B_1$ $9B_2$ $9B_3$ $9C_1$ $9C_2$ $9D_1$ $9D_2$
$3C$	73728	$6E_1$ $6K_3$ $6K_6$ $6L_1$ $6M_5$
	122880	$6E_4$ $6G_2$ $6K_1$ $6K_2$ $6K_5$ $6L_2$ $6L_3$ $6M_2$ $6M_4$ $15B_1$
	368640	$6E_2$ $6E_3$ $6G_1$ $6K_4$ $6K_7$ $6L_4$ $6M_1$ $6M_3$ $6M_6$ $15B_2$
$3D$	983040	$6J_2$ $6N_2$ $6O_1$ $9E_1$
	2949120	$6J_1$ $6N_1$ $6O_2$ $9E_2$
$4A$	103680	$4A'_2$ $12A_3$ $12J_1$ $20A_1$
	103680	$4A'_1$ $12A_4$ $12J_3$ $20A_2$
	414720	$4A'_4$ $12A_1$ $12A_5$ $12D_1$ $12J_4$ $12J_5$ $20A_4$ $28A_2$
	414720	$4A'_3$ $12A_2$ $12A_6$ $12D_2$ $12J_2$ $12J_6$ $20A_3$ $28A_1$
$4B$	103680	$8A_2$ $8B_4$
	103680	$8A_4$ $8B_5$
	103680	$8A_6$ $8B_2$
	414720	$8A_8$ $8B_10$ $12C_1$ $12G_4$
	1244160	$8A_9$ $8B_8$ $12C_2$ $12G_1$ $12G_2$ $12G_3$
	1658880	$8A_7$ $8A_10$ $8B_1$ $8B_3$ $8B_6$ $12B_1$ $12B_2$ $12E_1$ $12E_3$ $12G_5$ $12H_1$ $12H_2$ $12M_1$ $12M_2$
	1658880	$8A_1$ $8A_3$ $8A_5$ $8B_7$ $8B_9$ $12B_3$ $12B_4$ $12C_3$ $12E_2$ $12E_4$ $12H_3$ $12H_4$ $12M_3$ $12M_4$
$4C$	103680	$(abab^2)^2abab(ab^2)^4(ab)^5(ab^2)^2(ab)^4a$
	103680	$4C'_1$
	207360	$4C'_4$ $12I_1$
	207360	$4C'_3$ $12I_2$
	414720	$4C'_6$ $12N_1$
	414720	$4C'_5$ $12N_2$
	1244160	$4C_10'$ $12I_8$ $12N_5$ $20B_2$
	1244160	$4C_11'$ $12I_5$ $12I_6$
	1244160	$4C_12'$ $12I_9$ $12N_4$ $20B_3$
	1244160	$4C'_7$ $12I_7$ $12N_6$ $20B_1$
	1244160	$4C'_8$ $12I_3$ $12I_4$
	1244160	$4C'_9$ $12I_10$ $12N_3$ $20B_4$
$4D$	1244160	$4D'_2$
	1244160	$(ba)^5b^2ababab^2(ab)^3ab^2a$
	9953280	$8C_1$ $8C_2$ $12F_1$ $12F_2$ $12K_1$ $12K_2$ $12K_3$ $12K_4$ $12L_1$ $12L_2$ $12L_3$ $12L_4$ $12O_1$ $12O_2$ $12O_3$ $12O_4$
$5A$	1327104	$10A_1$ $10B_2$ $10B_3$ $10C_1$ $15A_1$
	6635520	$10A_4$ $10B_4$ $10B_6$ $10C_3$ $15A_2$ $15B_1$ $35A_1$
	6635520	$10A_2$ $10A_3$ $10B_1$ $10B_5$ $10C_2$ $10C_4$ $15A_3$ $15B_2$
$6A$	69120	$6A'_2$ $30B_3$

	69120	$6A_1'$ $30B_4$
	276480	$30B_1$ $30B_2$ $42A_1$ $42A_2$
$6B$	1474560	$18A_1$ $18A_2$ $18B_1$ $18B_2$ $18B_3$ $18B_4$
$6C$	110592	$12A_1$ $12I_2$ $12I_6$
	138240	$12A_4$ $12I_1$ $12I_7$
	414720	$6C_4'$ $30A_1$
	414720	$6C_3'$ $30A_2$
	829440	$12A_2$ $12A_6$ $12I_3$ $12I_4$ $12I_9$ $30A_4$
	1658880	$12A_3$ $12A_5$ $12I_5$ $12I_8$ $12I_10$ $30A_3$
$6D$	2211840	$12D_2$ $18E_2$ $18E_3$ $18E_4$
	2211840	$12D_1$ $18E_1$ $18F_1$ $18F_2$
$6E$	1105920	$6E_2'$
	1105920	$ab(ab^2)^2(ab)^3ab^2(ababab^2)^2a$
	2211840	$bababab^2ab^2ab^2abababab^2$
	2211840	$6E_3'$
$6F$	6635520	$12B_1$ $12B_2$ $12B_3$ $12B_4$ $12F_1$ $12F_2$
$6G$	3317760	$6G_2'$ $30C_1$
	3317760	$6G_1'$ $30C_2$
$6H$	13271040	$12E_1$ $12E_2$ $12E_3$ $12E_4$ $12K_1$ $12K_2$ $12K_3$
		$12K_4$ $18C_1$ $18C_2$ $18D_1$ $18G_1$ $18G_2$
$6I$	414720	$12C_1$ $12G_3$
	829440	$6I_3'$
	829440	$ababab^2ab^2(ab)^4(ab^2)^3ab(ab^2)^2ababa$
	1244160	$12C_2$ $12G_1$ $12G_2$ $12G_4$
	1658880	$bab^2ababab^2abababab^2abababa$
	1658880	$6I_5'$
	9953280	$12C_3$ $12G_5$
$6J$	8847360	$6J_2'$ $18H_2$
	8847360	$6J_1'$ $18H_1$
$6K$	1105920	$12J_4$ $12N_2$
	2211840	$12J_3$ $12N_1$ $12N_6$
	3317760	$12J_5$ $12N_5$
	3317760	$12J_1$ $12N_3$
	3317760	$6K_6'$
	3317760	$abababab^2$
	6635520	$12J_2$ $12J_6$ $12N_4$
$6L$	3317760	$6L_2'$
	3317760	$abab^2ab^2abababab^2ababab^2ab^2a$
	6635520	$12H_1$ $12H_2$ $12L_2$ $12L_4$
	19906560	$12H_3$ $12H_4$ $12L_1$ $12L_3$
$6M$	3317760	$bab^2abab^2ab^2ab^2abababab^2a$
	3317760	$6M_1'$
	6635520	$6M_4'$
	6635520	$abab^2ababababababab^2$
	9953280	$6M_6'$
	9953280	$ab^2ab^2ab^2ababababababab^2$
$6N$	26542080	$bababab^2ababab^2a$
	26542080	$6N_1'$
$6O$	26542080	$12M_1$ $12M_2$ $12O_1$ $12O_2$

	79626240	$12M_3$ $12M_4$ $12O_3$ $12O_4$
$7A$	2654208	$14A_2$ $14B_2$
	26542080	$14A_1$ $14A_3$ $14B_4$ $21A_1$
	39813120	$14A_4$ $14B_1$ $14B_3$ $21A_2$ $35A_1$
$8A$	3317760	$8A'_4$
	3317760	$8A'_3$
	3317760	$bababab^2abababab^2ababab^2abab^2a$
	3317760	$babab^2ab^2ab^2ab^2abababababab^2$
	9953280	$8A'_6$
	9953280	$babab^2ababababababab^2a$
	19906560	$8A'_8$ $24C_3$
	19906560	$8A'_7$ $24C_4$
	19906560	$8A_10'$ $24C_2$
	19906560	$8A'_9$ $24C_1$
$8B$	3317760	$8B'_2$
	3317760	$abababab^2abababababab^2ab^2ababab^2$
	9953280	$8B'_4$
	9953280	$bab^2ab^2ababababababababab^2$
	9953280	$bababab^2ababababab^2$
	9953280	$8B'_5$
	19906560	$8B_10'$ $24A_4$
	19906560	$8B'_9$ $24A_3$
	19906560	$8B'_8$ $24A_2$
	19906560	$8B'_7$ $24A_1$
$8C$	39813120	$8C'_2$ $24B_1$ $24B_3$
	39813120	$8C'_1$ $24B_2$ $24B_4$
$9A$	17694720	$18D_1$ $27A_1$
$9B$	4423680	$18B_1$ $18E_1$ $18E_2$
	13271040	$18B_4$ $18E_3$
	26542080	$18B_2$ $18B_3$ $18E_4$
$9C$	8847360	$18A_2$ $18C_1$ $18F_1$
	26542080	$18A_1$ $18C_2$ $18F_2$
$9D$	17694720	$18G_2$
	53084160	$18G_1$
$9E$	53084160	$18H_1$
	159252480	$18H_2$
$10A$	13271040	$10A'_2$ $30B_3$
	13271040	$10A'_1$ $30B_4$
	19906560	$10A'_4$ $30B_1$
	19906560	$10A'_3$ $30B_2$
$10B$	13271040	$20A_1$ $20B_4$
	13271040	$20A_2$ $20B_1$
	19906560	$10B'_4$ $30A_1$
	19906560	$10B'_3$ $30A_2$
	39813120	$20A_3$ $20B_3$ $30A_4$
	39813120	$20A_4$ $20B_2$ $30A_3$
$10C$	19906560	$10C'_2$
	19906560	$bababab^2abababab^2ab^2a$
	39813120	$10C'_4$ $30C_2$

	39813120	$10C_3'\ 30C_1$
$11A$	159252480	$22A_1$
$12A$	3317760	$12A_2'$
	3317760	$abab^2abababababab^2ababab^2a$
	3317760	$12A_4'$
	3317760	$bab^2abababab^2ababab^2$
	9953280	$12A_6'\ 60A_2$
	9953280	$12A_5'\ 60A_1$
$12B$	6635520	$12B_2'$
	6635520	$bab^2abababababababab^2$
	19906560	$12B_4'$
	19906560	$abab^2abababababab^2abab^2a$
$12C$	3317760	$24A_1$
	9953280	$24A_3$
	39813120	$24A_2\ 24A_4$
$12D$	26542080	$12D_2'\ 36B_1$
	26542080	$12D_1'\ 36B_2$
$12E$	13271040	$12E_3'\ 36A_2$
	13271040	$12E_4'$
	13271040	$12E_1'\ 36A_1$
	13271040	$abababab^2ab^2ababab^2$
$12F$	39813120	$24B_2\ 24B_3$
	39813120	$24B_1\ 24B_4$
$12G$	9953280	$12G_3'$
	9953280	$24C_2$
	9953280	$babab^2ababab^2ababab^2$
	9953280	$24C_4$
	39813120	$24C_1\ 24C_3$
$12H$	13271040	$12H_2'$
	13271040	$bab^2ab^2ababab^2ab^2abababab^2a$
	39813120	$12H_4'$
	39813120	$bab^2ab^2abababa$
$12I$	3317760	$12I_2'$
	3317760	$abababab^2abababab^2ab^2ababab^2$
	9953280	$abababab^2abababababab^2abab^2$
	9953280	$abababab^2ababab^2ab^2ab^2$
	9953280	$12I_3'$
	9953280	$12I_4'$
	19906560	$12I_8'$
	19906560	$babababababababab^2$
	19906560	$12I_10'$
	19906560	$babababababab^2ab^2ab^2a$
$12J$	13271040	$12J_3'$
	13271040	$12J_4'$
	13271040	$bababab^2ababababab^2abab^2a$
	13271040	$ababab^2ababab^2ab^2ababab^2a$
	39813120	$abababab^2abab^2$
	39813120	$12J_5'$
$12K$	39813120	$bababab^2ababab^2ababab^2$

	39813120	$12K_3'$
	39813120	$bab^2ab^2ab^2ab^2ababababababa$
	39813120	$12K_1'$
$12L$	39813120	$bab^2ab^2abababab^2abab^2a$
	39813120	$bab^2ab^2ab^2abababab^2a$
	39813120	$12L_2'$
	39813120	$12L_1'$
$12M$	26542080	$abab^2ababababababab^2a$
	26542080	$12M_1'$
	79626240	b
	79626240	$12M_3'$
$12N$	13271040	$babababababab^2$
	13271040	$12N_1'$
	39813120	$bab^2ab^2abab^2ababab^2a$
	39813120	$12N_3'$
	39813120	$abab^2ababababababab^2$
	39813120	$12N_5'$
$12O$	79626240	$12O_3'$
	79626240	$ababab^2ababab^2$
	79626240	$bab^2abababab^2ab^2ab^2a$
	79626240	$12O_2'$
$13A$	53084160	$26A_2$
	159252480	$26A_1$ $39A_1$
$13B$	53084160	$26B_2$
	159252480	$26B_1$ $39B_1$
$14A$	26542080	$bab^2ababab^2ab^2abababababa$
	26542080	$14A_1'$
	79626240	$14A_4'$ $42A_2$
	79626240	$14A_3'$ $42A_1$
$14B$	39813120	$bababab^2ababababababababa$
	39813120	$14B_1'$
	79626240	$28A_2$
	79626240	$28A_1$
$15A$	13271040	$30A_1$ $30B_3$
	39813120	$30A_2$ $30A_3$ $30B_2$
	79626240	$30A_4$ $30B_1$ $30B_4$
$15B$	53084160	$30C_2$
	159252480	$30C_1$
$17A$	318504960	$bababab^2ab^2abab^2$
$18A$	26542080	$18A_2'$
	26542080	$babab^2ab^2ab^2ababab^2$
$18B$	26542080	$bababababab^2$
	26542080	$18B_1'$
	79626240	$babab^2abababab^2$
	79626240	$18B_3'$
$18C$	79626240	$babababab^2ab^2a$
	79626240	$18C_1'$
$18D$	159252480	$36A_1$ $36A_2$
$18E$	26542080	$36B_1$

	39813120	$bababab^2abababababa$
	39813120	$18E_2'$
	79626240	$36B_2$
$18F$	26542080	$abababab^2ab^2abab^2$
	79626240	$ababababababab^2ab^2ababab^2$
$18G$	159252480	$bab^2abababa ba$
	159252480	$18G_1'$
$18H$	159252480	$bab^2ababab^2$
	159252480	$18H_1'$
$20A$	26542080	$bab^2abababab^2ab^2abababab^2$
	26542080	$20A_1'$
	79626240	$20A_4'$ $60A_1$
	79626240	$20A_3'$ $60A_2$
$20B$	79626240	$20B_2'$
	79626240	$bababab^2ab^2a$
	79626240	$bababab^2abababab^2$
	79626240	$20B_3'$
$21A$	159252480	$42A_2$
	159252480	$42A_1$
$22A$	159252480	$ababab^2abababab^2$
$24A$	79626240	$24A_4'$
	79626240	$24A_3'$
	79626240	$babababab^2a$
	79626240	$ababababab^2ab^2ab^2$
$24B$	79626240	$24B_2'$
	79626240	$bab^2ababab^2abababab^2a$
	79626240	$bababab^2a$
	79626240	$24B_3'$
$24C$	79626240	$24C_2'$
	79626240	$bab^2ababababab^2$
	79626240	$bababababab ab^2$
	79626240	$24C_3'$
$26A$	159252480	$bababab^2$
	159252480	$26A_1'$
$26B$	159252480	$ababab^2ababababab^2$
	159252480	$26B_1'$
$27A$	318504960	$bababab^2ab^2$
$28A$	159252480	$abab^2ababababab^2$
	159252480	$28A_1'$
$30A$	39813120	$30A_2'$
	39813120	$babababab ab^2abab^2a$
	79626240	$60A_2$
	79626240	$60A_1$
$30B$	79626240	$abab^2abababab^2$
	79626240	$30B_1'$
	79626240	$babababab^2ab^2abab^2$
	79626240	$30B_3'$
$30C$	159252480	$30C_2'$
	159252480	$bab^2ababab^2ababab^2$

$35A$	318504960	$bababababab^2a$
$36A$	159252480	$36A_2'$
	159252480	$bab^2ab^2ababa ba$
$36B$	159252480	$36B_2'$
	159252480	$abab^2ababab^2$
$39A$	318504960	$bab^2abababab^2$
$39B$	318504960	$babab^2$
$42A$	159252480	$bababababab^2ab^2a$
	159252480	$42A_1'$
$60A$	159252480	$babababab^2$
	159252480	$60A_1'$

Table 9: $G \cong Fi_{24}{}'$, $X = 2A$, $t = a$.

C	Orbit size	Representative
$1A$	1	-
$2A$	720	$4B_2$, $6F_1$
	123552	$4B_2$, $6A_1$
$2B$	1216215	$4A_1$, $6A_1$
$3A$	56320	$6A_1$
$3C$	20500480	$6F_1$
$3E$	60825600	$(ba)^3b^2(ab)^3$
$4A$	389188800	$(ba)^4b$
$4B$	88957440	$(b^2a)^2ba(b^2a)^2b(ab)^3$
	88957440	$4B_1'$
$5A$	1423319040	b^2ab
$6A$	19768320	$b^2(ab)^5(ab^2)^2ab$
$6F$	2767564800	bab

Table 10: $G \cong Fi_{24}'$, $t \in X = 2B$

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
t	1	1	Δ_0
$2A_1$	24192	$2^7 \cdot 3^3 \cdot 7$	Δ_1
$2A_2$	45360	$2^4 \cdot 3^4 \cdot 5 \cdot 7$	Δ_1
$2B_1$	3402	$2 \cdot 3^5 \cdot 7$	Δ_1
$2B_2$	816480	$2^5 \cdot 3^6 \cdot 5 \cdot 7$	Δ_1
$2B_3$	2177280	$2^8 \cdot 3^5 \cdot 5 \cdot 7$	Δ_1
$3A_1$	258048	$2^{12} \cdot 3^2 \cdot 7$	Δ_2
$3B_1$	917504	$2^{17} \cdot 7$	Δ_2
$3C_1$	1032192	$2^{14} \cdot 3^2 \cdot 7$	Δ_2
$3C_2$	10321920	$2^{15} \cdot 3^2 \cdot 5 \cdot 7$	Δ_2
$3D_1$	165150720	$2^{19} \cdot 3^2 \cdot 5 \cdot 7$	Δ_2
$3E_1$	278691840	$2^{15} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$4A_1$	1306368	$2^8 \cdot 3^6 \cdot 7$	Δ_2
$4A_2$	4354560	$2^9 \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$4A_3$	104509440	$2^{12} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$4A_4$	139345920	$2^{14} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$4B_{1,2}$	52254720	$2^{11} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$4B_{3,4}$	209018880	$2^{13} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$4C_{1,2}$	78382080	$2^{10} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$4C_{3,4,5}$	313528320	$2^{12} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$4C_6$	1254113280	$2^{14} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$5A_1$	334430208	$2^{16} \cdot 3^6 \cdot 7$	Δ_2
$5A_2$	836075520	$2^{15} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$6A_1$	34836480	$2^{12} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6A_2$	55738368	$2^{15} \cdot 3^5 \cdot 7$	Δ_2
$6A_3$	69672960	$2^{13} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6B_1$	278691840	$2^{15} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6C_1$	123863040	$2^{17} \cdot 3^3 \cdot 5 \cdot 7$	Δ_2
$6C_2$	371589120	$2^{17} \cdot 3^4 \cdot 5 \cdot 7$	Δ_2
$6D_1$	209018880	$2^{13} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$6D_2$	836075520	$2^{15} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$6E_1$	1114767360	$2^{17} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6F_1$	139345920	$2^{14} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6F_{2,3,4}$	278691840	$2^{15} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6F_5$	1114767360	$2^{17} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6G_1$	4459069440	$2^{19} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6H_1$	4459069440	$2^{19} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$6I_{1,2,3}$	836075520	$2^{15} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$6I_4$	3344302080	$2^{17} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$6J_1$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$6K_1$	5016453120	$2^{16} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$6K_2$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2

Table 10 (cont.)

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$7A_1$	63700992	$2^{18} \cdot 3^5$	Δ_3
$7A_2$	6688604160	$2^{18} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$7B_{1,2}$	11466178560	$2^{20} \cdot 3^7 \cdot 5$	Δ_2
$8A_{1,2}$	278691840	$2^{15} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$8A_{3,4}$	836075520	$2^{15} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$8A_{5,6}$	2508226560	$2^{15} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$8A_{7,8}$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$8B_1$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$8B_2$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$8C_{1,2,3,4}$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$9A_1$	2972712960	$2^{20} \cdot 3^4 \cdot 5 \cdot 7$	Δ_2
$9B_1$	445906944	$2^{18} \cdot 3^5 \cdot 7$	Δ_3
$9B_2$	2229534720	$2^{18} \cdot 3^5 \cdot 5 \cdot 7$	Δ_3
$9C_1$	4459069440	$2^{19} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$9D_1$	5945425920	$2^{21} \cdot 3^4 \cdot 5 \cdot 7$	Δ_3
$9E_{1,2}$	8918138880	$2^{20} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$9F_1$	26754416640	$2^{20} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$10A_{1,2}$	5016453120	$2^{16} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$10A_3$	6688604160	$2^{18} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$10A_4$	668860416	$2^{17} \cdot 3^6 \cdot 7$	Δ_2
$10A_5$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$10B_{1,2,3}$	5016453120	$2^{16} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$10B_{4,5}$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$11A_1$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$12A_1$	1672151040	$2^{16} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12A_2$	3344302080	$2^{17} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12B_{1,2}$	1114767360	$2^{17} \cdot 3^5 \cdot 5 \cdot 7$	Δ_2
$12B_{3,4}$	3344302080	$2^{17} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12C_1$	5016453120	$2^{16} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12C_2$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12D_{1,2,3,4}$	1672151040	$2^{16} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12D_{5,6}$	5016453120	$2^{16} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12E_{1,2}$	3344302080	$2^{17} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12E_{3,4}$	6688604160	$2^{18} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12F_{1,2}$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12G_{1,2}$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12H_{1,2}$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12I_1$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12I_2$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12J_1$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12J_2$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12K_{1,2}$	10032906240	$2^{17} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12K_{3,4}$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12L_{1,2,3,4}$	6688604160	$2^{18} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$12L_{5,6}$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$12M_{1,2}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2

Table 10 (cont.)

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$13A_1$	8918138880	$2^{20} \cdot 3^5 \cdot 5 \cdot 7$	Δ_3
$13A_2$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$14A_{1,2}$	6688604160	$2^{18} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$14A_3$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$14A_4$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$14B_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$15A_{1,2}$	6688604160	$2^{18} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$15B_1$	53508833280	$2^{21} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$15C_{1,2}$	26754416640	$2^{20} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$16A_{1,2,3,4}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$17A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$18A_1$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$18B_{1,2}$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$18C_1$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$18D_{1,2}$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_3
$18D_{3,4}$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$18E_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$18F_1$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$20A_{1,2}$	13377208320	$2^{19} \cdot 3^6 \cdot 5 \cdot 7$	Δ_2
$20A_{3,4}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$20B_{1,2,3,4,5,6}$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$21A_1$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$21B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$21C_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$21D_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$22A_1$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$24A_{1,2}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$24B_{1,2}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$24C_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$24D_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$24E_{1,2,3,4}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$24F_{1,2,3,4}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$24G_{1,2,3,4}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$26A_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$27A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$27B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$27B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$28A_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$29A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$29B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$30A_{1,2}$	20065812480	$2^{18} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$30A_{3,4}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$30B_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$33A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$33B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3

Table 10 (cont.)

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$35A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$36A_{1,2}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$36B_{1,2}$	40131624960	$2^{19} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$36C_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$36D_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$39A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$39B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$39C_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$39D_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$42A_1$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$42B_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$42C_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_2
$45A_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$45B_1$	160526499840	$2^{21} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3
$60A_{1,2}$	80263249920	$2^{20} \cdot 3^7 \cdot 5 \cdot 7$	Δ_3

Table 11: $G \cong Fi_{24}$, $t \in X = 2D$

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$1A_1$	1	1	Δ_0
$2A_1$	2079	$3^3 \cdot 7 \cdot 11$	Δ_1
$2A_2$	38016	$2^7 \cdot 3^3 \cdot 11$	Δ_1
$2A_3$	62370	$2 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11$	Δ_1
$2B_1$	187110	$2 \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_1
$2B_2$	997920	$2^5 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11$	Δ_1
$2B_3$	2395008	$2^7 \cdot 3^5 \cdot 7 \cdot 11$	Δ_1
$3A_1$	8448	$2^8 \cdot 3 \cdot 11$	Δ_2
$3A_2$	221760	$2^6 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$3B_1$	1261568	$2^{14} \cdot 7 \cdot 11$	Δ_2
$3C_1$	1892352	$2^{13} \cdot 3 \cdot 7 \cdot 11$	Δ_2
$3C_2$	17031168	$2^{13} \cdot 3^3 \cdot 7 \cdot 11$	Δ_2
$3D_1$	37847040	$2^{15} \cdot 3 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$3D_2$	113541120	$2^{15} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$3E_1$	48660480	$2^{15} \cdot 3^3 \cdot 5 \cdot 11$	Δ_2
$4A_1$	23950080	$2^8 \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4A_2$	57480192	$2^{10} \cdot 3^6 \cdot 7 \cdot 11$	Δ_2
$4A_3$	71850240	$2^8 \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4A_4$	95800320	$2^{10} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4B_{1,2}$	4790016	$2^8 \cdot 3^5 \cdot 7 \cdot 11$	Δ_2
$4B_{3,4}$	23950080	$2^8 \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4B_{5,6}$	71850240	$2^8 \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4B_{7,8}$	287400960	$2^{10} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4C_1$	287400960	$2^{10} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$4C_2$	574801920	$2^{11} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$4C_3$	1724405760	$2^{11} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$5A_1$	43794432	$2^{14} \cdot 3^5 \cdot 11$	Δ_2
$5A_2$	1532805120	$2^{14} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6A_{1,2}$	2128896	$2^{10} \cdot 3^3 \cdot 7 \cdot 11$	Δ_2
$6A_3$	9123840	$2^{11} \cdot 3^4 \cdot 5 \cdot 11$	Δ_3
$6A_{4,5}$	23950080	$2^8 \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6A_{6,7}$	95800320	$2^{10} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6B_{1,2}$	153280512	$2^{13} \cdot 3^5 \cdot 7 \cdot 11$	Δ_2
$6C_1$	170311680	$2^{14} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6C_2$	510935040	$2^{14} \cdot 3^4 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6D_1$	23950080	$2^8 \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6D_2$	35925120	$2^7 \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6D_{3,4,5,6}$	95800320	$2^{10} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6D_7$	574801920	$2^{11} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6E_1$	1532805120	$2^{14} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6F_{1,2}$	85155840	$2^{13} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6F_{3,4,5,6}$	766402560	$2^{13} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$6G_1$	2043740160	$2^{16} \cdot 3^4 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$6G_{2,3}$	3065610240	$2^{15} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2

Table 11 (cont.)

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$6H_1$	2043740160	$2^{16}\cdot 3^4\cdot 5\cdot 7\cdot 11$	Δ_3
$6H_{2,3,4}$	3065610240	$2^{15}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$6H_5$	9196830720	$2^{15}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$6I_{1,2,3,4,5}$	766402560	$2^{13}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$6I_6$	6897623040	$2^{13}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$6K_1$	3065610240	$2^{15}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$7A_1$	1839366144	$2^{15}\cdot 3^6\cdot 7\cdot 11$	Δ_2
$7A_2$	9196830720	$2^{15}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$7B_1$	10510663680	$2^{18}\cdot 3^6\cdot 5\cdot 11$	Δ_3
$8A_{1,2,3,4}$	2299207680	$2^{13}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$8A_{5,6,7,8}$	6897623040	$2^{13}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$8B_1$	27590492160	$2^{15}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$8C_{1,2}$	9196830720	$2^{15}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_3
$9A_1$	1362493440	$2^{17}\cdot 3^3\cdot 5\cdot 7\cdot 11$	Δ_2
$9B_1$	340623360	$2^{15}\cdot 3^3\cdot 5\cdot 7\cdot 11$	Δ_2
$9B_2$	3065610240	$2^{15}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$9C_1$	340623360	$2^{15}\cdot 3^3\cdot 5\cdot 7\cdot 11$	Δ_2
$9C_2$	3065610240	$2^{15}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$9D_1$	2724986880	$2^{18}\cdot 3^3\cdot 5\cdot 7\cdot 11$	Δ_2
$9E_1$	1362493440	$2^{17}\cdot 3^3\cdot 5\cdot 7\cdot 11$	Δ_2
$9E_2$	12262440960	$2^{17}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$9F_1$	6131220480	$2^{16}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_3
$9F_2$	12262440960	$2^{17}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_3
$9F_3$	55180984320	$2^{16}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_3
$10A_1$	919683072	$2^{14}\cdot 3^6\cdot 7\cdot 11$	Δ_2
$10A_{2,3}$	4598415360	$2^{14}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$10A_4$	9196830720	$2^{15}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$10A_5$	13795246080	$2^{14}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$10B_{1,2}$	4598415360	$2^{14}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$10B_3$	27590492160	$2^{15}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$11A_1$	110361968640	$2^{17}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$12A_1$	191600640	$2^{11}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$12A_2$	574801920	$2^{11}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$12A_3$	4598415360	$2^{14}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$12B_{1,2}$	1532805120	$2^{14}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$12B_{3,4}$	4598415360	$2^{14}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$12C_1$	574801920	$2^{11}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$12C_2$	1149603840	$2^{12}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_3
$12C_{3,4,5}$	1724405760	$2^{11}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$12C_6$	4598415360	$2^{14}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$12D_{1,2}$	153280512	$2^{13}\cdot 3^5\cdot 7\cdot 11$	Δ_2
$12D_{3,4,5,6}$	2299207680	$2^{13}\cdot 3^6\cdot 5\cdot 7\cdot 11$	Δ_2
$12D_{7,8}$	6897623040	$2^{13}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2
$12E_1$	3065610240	$2^{15}\cdot 3^5\cdot 5\cdot 7\cdot 11$	Δ_2
$12E_2$	27590492160	$2^{15}\cdot 3^7\cdot 5\cdot 7\cdot 11$	Δ_2

Table 11 (cont.)

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$12F_{1,2}$	3065610240	$2^{15} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12F_{3,4}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12G_{1,2}$	18393661440	$2^{16} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12H_{1,2}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12J_{1,2}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12K_{1,2}$	3065610240	$2^{15} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12K_{3,4,5,6}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12K_{7,8}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$12L_{1,2}$	18393661440	$2^{16} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$12L_{3,4,5,6}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$13A_{1,2}$	36787322880	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$14A_1$	18393661440	$2^{16} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$14A_{2,3}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$14A_4$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$14B_1$	73574645760	$2^{18} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$15A_1$	613122048	$2^{15} \cdot 3^5 \cdot 7 \cdot 11$	Δ_2
$15A_2$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$15B_1$	73574645760	$2^{18} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$15C_1$	12262440960	$2^{17} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$15C_2$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$17A_1$	220723937280	$2^{18} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$18A_{1,2}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18B_1$	3065610240	$2^{15} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18B_{2,3}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18B_4$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18C_1$	36787322880	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18D_1$	6131220480	$2^{16} \cdot 3^5 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18D_{2,3}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18D_4$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18E_{1,2}$	36787322880	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$18F_1$	36787322880	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$18F_{2,3}$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$18G_{1,2}$	36787322880	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$18G_{3,4}$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$20A_{1,2}$	18393661440	$2^{16} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$20A_{3,4}$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$20B_1$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$21A_1$	18393661440	$2^{16} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$21A_2$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$21B_1$	73574645760	$2^{18} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$22A_1$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$24A_{1,2}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$24A_{3,4}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$24B_{1,2}$	9196830720	$2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$24B_{3,4}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$24D_{1,2,3,4}$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2

Table 11 (cont.)

$C_G(t)$-orbit	Orbit Size	(factored)	Disk
$26A_{1,2}$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$27A_1$	73574645760	$2^{18} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$28A_{1,2}$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$30A_{1,2}$	27590492160	$2^{15} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$30A_{3,4}$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$30B_{1,2}$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$33A_1$	220723937280	$2^{18} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$33B_1$	220723937280	$2^{18} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$35A_1$	220723937280	$2^{18} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$36B_{1,2}$	36787322880	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$36C_{1,2}$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2
$39A_{1,2}$	73574645760	$2^{18} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$42A_{1,2}$	55180984320	$2^{16} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$42A_3$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_3
$60A_{1,2}$	110361968640	$2^{17} \cdot 3^7 \cdot 5 \cdot 7 \cdot 11$	Δ_2

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