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Product Scheduling for Thermal Energy Reduction in Papermaking Industries ⋆

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Abstract: Papermaking is considered as an energy-intensive industry partly due to the fact that the machinery and procedures have been designed at the time when energy was both cheap and plentiful. A typical paper machine manufactures a variety of different products (grades) which impose variable per-unit raw material and energy costs to the mill. It is known that during a grade change operation the products are not market-worthy. Therefore, two different production regimes, i.e. steady state and grade transition can be recognised in papermaking practice. Among the costs associated with paper manufacture, the energy cost is 'more variable' due to (usually) day-to-day variations of the energy prices. Moreover, the production of a grade is often constrained by customer delivery time requirements. Given the above constraints and production modes, the product scheduling technique proposed in this paper aims at optimising the sequence of orders in a single machine so that the cost of production (mainly determined by the energy) is minimised. Simulation results obtained from a commercial board machine in the UK confirm the effectiveness of the proposed method.

Keywords: Single machine scheduling, optimisation, paper machine, energy reduction

1. INTRODUCTION

During recent years, reducing production costs has become an important success factor for manufacturing industries. This is due to the rising costs of energy and tighter financial restrictions. In this context, it is important for manufacturing industries to efficiently utilise their resources (e.g. energy, machinery and staff) while satisfying customer requirements (e.g. required quality and/or on-time delivery of the final products). These objectives often impose certain policies for optimised production planning. In a paper manufacturing plant, usually a number of different products (paper grades) are produced during a run of the paper machine. Producing each paper grade incorporates a nominal raw material (fibres and chemicals) and energy (thermal or electrical) consumption which contribute to the production costs. It is known that the energy prices have the most notable (almost daily) variations amongst the elements affecting the production costs. Furthermore, each customer order is often constrained by a delivery deadline. Considering the collection of above mentioned issues, it is important for paper makers to schedule the production so that the resource allocation and customer satisfaction are optimised simultaneously. The ultimate goal of such planning is improving the overall profitability, which can be realised by reducing costs (energy and raw material usage, plant downtime, etc.) or increasing the productivity. As for papermaking industry, the main determinant of the production cost is that of energy consumption. Therefore, aiming to optimise the energy usage in a paper machine scheduling must be taken into consideration which is the main focus of this paper.

Different production scheduling techniques are usually distinguished from each other by the assumptions made for the allocation of machinery in a manufacturing line and their interconnections. In this regard, approaches such as reinforcement learning have been employed for multi-machine scheduling (Choi et al. (2007)). As for single-machine scheduling, minimising the total makespan has been taken into account, considering Branch and Bound (Baptiste and Le Pape (2005)) or Linear Programming (LP) algorithms (Chou et al. (2009)). Single-machine makespan minimisation has been also studied by Margot et al. (2003) where LP relaxation is applied to precedence constrained single-machine scheduling problem. Furthermore, Model Predictive Control (MPC) has been applied to scheduling problems in production lines. For instance, Vargas-Villamil and Rivera (1997) have formulated long term time and machine allocation in semiconductor manufacturing facilities as a state-space and applied an L_1 norm finite moving horizon cost function to determine the

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optimal scheduling law. Also, from a theoretical point of view, De Schutter and van den Boom (2003) have considered a class of discrete-event systems with both hard and soft synchronization constraints, leading to bilinear $(Max, +)$ algebraic models and an extended MPC framework for product scheduling. Recently, Tang et al. (2010) have applied MPC to solve the scheduling problems for parallel machines with processing and completion time uncertainties.

In majority of mentioned scheduling techniques, the principal objective has been minimising the makespan or the costs linked with delivery delays. In this context, less attention has been paid to economic aspects of scheduling in terms of production costs. In particular, limited research has been carried out to address two important issues in multi-product scheduling approaches, namely, the cost of production during a grade change and more importantly, the varying costs of energy. Therefore, the is a need to embed economic aspects of energy consumption in product scheduling in manufacturing industries.

This paper aims at addressing the mentioned gaps by proposing a single-machine, multi-product scheduling scheme for papermaking. The objective here is to determine an optimal production policy so that the energy and raw material costs related to both steady-state and grade transition operations are minimised, while the customer requirements (here, on time delivery) are met. Fig. 1 demonstrates the scheme of the proposed scheduling method. As the figure suggests, product scheduling task

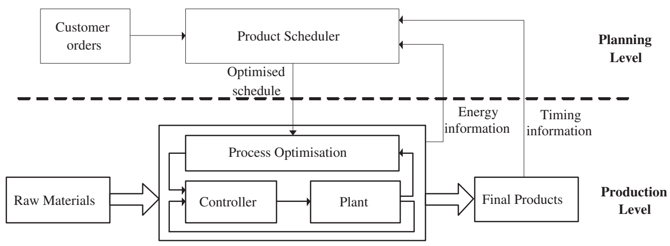

Fig. 1. Scheme of production scheduling

is considered a high-level activity compared to factory-level process optimisation and control tasks. It is shown in this paper that this can be formulated as a binary, semi-definite programming problem with quadratic constraints. In the next section, an overview of product scheduling in papermaking is proposed.

2. PRODUCT SCHEDULING IN PAPERMAKING

In papermaking, different customer orders (product grades) are often different in their basis weight. Therefore, 'grade change' operation must be performed by machine operators whenever a new grade is planned for production. Different grades also demand using different amount of energy, raw materials and machine settings. It is assumed here that maximum profitability is achieved by maximum operational efficiency, i.e. by reducing the raw material and more importantly, the steam consumption costs. From a production planning point of view, the optimisation of the sequence of events (customer orders) for a maximised profitability can be considered as a Discrete Event System (DES) optimisation problem. However, from a shop-floor point of view, a grade change has certain time-driven dynamics as it is in fact a product quality change which requires changing several inputs to the machine. Therefore, broadly speaking, hybrid dynamics should be considered to model the scheduling problem in multi-grade machines. However, unlike (Tousain and Bosgra (2006)), only the transition time caused by the grade change dynamics is considered in this paper.

The production during a grade transition is not market-worthy as the dynamics of the grade changes are highly nonlinear and there are not sufficient control tools for nonlinear grade change operations (Jokinen and Ritala (2010)). In addition, the chemical state of wet-end of the paper machine is usually different from a paper grade to another. This implies that chemicals retained from previous grades in the wet-end might have detrimental effect for the quality of the successive grades. Therefore, during a grade change (*grade transition time*), a preparatory chemical treatment phase might be necessary at the wet-end which would incorporate a preparation time between the two grades. The length of grade transition highly depends on the operators' expertise and can vary from 10 to 35 minutes in a board machine. As it can be implied, the longer time constants associated with the nonlinear time driven dynamics would result in less productivity, less energy efficiency, and in extreme cases, in failing to meet the delivery deadline. Moreover, a grade change operation incorporates a risk of paper breaks at both wet and dry ends of the machine which, if occurred, would impose costs in terms of raw material loss and recovery time. As a result, the common practice in paper manufacture scheduling is to changes the grades by open loop 'ramping cycle' of the basis weight. That is, keeping the basis weight set-point changes as small as possible which does not necessarily lead to optimised energy usage or delivery time. The approach proposed here aims at scheduling with optimised energy and raw material efficiency during both steady-state and grade change periods.

3. PROBLEM FORMULATION

In this section, the product scheduling problem introduced in sections 1 and 2 is formally defined.

3.1 Product Scheduling Problem

Suppose that the paper machine is required to manufacture m different orders of n paper grades ($m \leq n$). Note τ_j as j^{th} grade's processing time (i.e. the time required to produce grade j). Therefore, for the n different grades, the processing time vector $T(i)$ is defined as,

$$T(i) = [\tau(i,1), \cdots, \tau(i,j), \cdots, \tau(i,n)], \tag{1}$$

where $\tau(j) = \frac{O(j)}{P(j)}$ with $O(j)$ denotes the total amount of grade j (ordered in tonnes) and $P(j)$ is the production rate set for the machine during production of grade j (in $\frac{tonnes}{hr}$). Furthermore, in order to consider the transitional state of the paper machine during the grade change, a grade transition time $\mu(j,k)$ is defined as the time required to change the production from grade j to arbitrary grade k. Here, it is assumed that the transition time is not reversible, i.e. $\mu(j,k) \neq \mu(k,j)$ and also no time is required to produce the same grade after completion of the order corresponding to j^{th} grade, $(\mu(j,j) = 0)$. Fig. 2 illustrates the concept of grade change with grade transition effect in a UK-based commercial board machine.

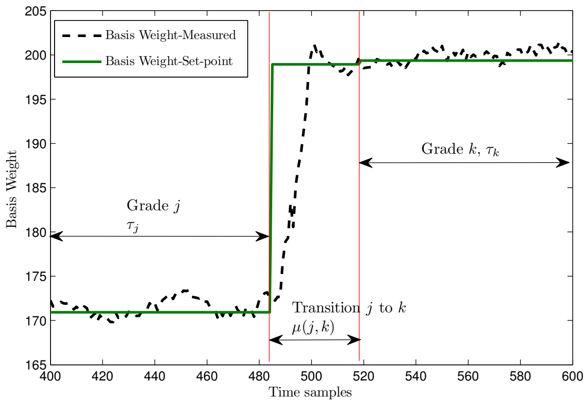

Fig. 2. Illustration of a grade change and associated times

In addition, consider $\Pi(i),\ (i = 1, 2, \cdots, m)$ as the binary policy herein called scheduling vector which determines the production slot of the i^{th} order.

$$\Pi(i) = [\pi(i,1)\ \cdots\ \pi(i,n)] \tag{2}$$

The scheduling vector is assumed to satisfy the following constraints,

$$\pi(i,j) \in \{0,1\}\ \forall i,j, \tag{3}$$
$$\|\Pi(i)\|_0 = 1, \forall i, \tag{4}$$
$$\Pi(i) \times \Pi^T(j) = 0,\ \forall i \neq j, \tag{5}$$
$$T_p \times \Pi^T(i) \leq T_d. \tag{6}$$

The constraint (3) is to determine the grade scheduled for production, while (4) is to ensure that only one non-zero element exists in each row of scheduling matrix Π, that is, only one grade is manufactured at a time. Furthermore, (5) is to guarantee the mutual exclusiveness of the orders. That is, only one particular order can be processed at a time. Also, given the customer's requirement (weight of the paper grade requested) and machine's nominal production rate for corresponding grades, constraint (6) ensures that selected grade is delivered before the allocated delivery deadline. The diagonal, $n \times n$ matrix T_p consists of the total production time for each grade as follows,

$$T_p(j) = \sum_{q=1}^{j-1} \tau(q) + \mu(q-1,q) + \tau(j), \tag{7}$$

where T_d is an $(m \times 1)$ vector formed of the delivery deadlines agreed with the customer.

3.2 Setting the Optimisation Problem

As noted earlier, the costs are limited to those of steady state and grade transition. The probabilistic cost associated with the risk of paper breaks (caused by grade change operations) is not covered here.

Steady-State Production Costs: This is a cost inherent in the production of each grade and consists of energy use, raw materials and chemicals . Often the cost of energy has the most significant variations compared to the raw material and chemicals. Noting the steady-state production cost of the i^{th} order as $C_p(i,j)$, it can be modelled as follows,

$$C_p(i,j) = \int_0^{\tau_j} \Big(E(t,i,j) + R(t,i,j)\Big)dt, \tag{8}$$

where $E(t,i,j),\ R(t,i,j)$ are continuous signals denoting the rates of energy and raw material consumption, respectively. These must be either measured or estimated using a set of relevant process measurements. In this paper, measurable signals 'steam flow' and 'retention aids flow' have been considered as the measures to model energy and raw material consumption.

Transitional Costs: As mentioned in section 2, during a grade change, the product is not financially viable. Therefore, the energy and material consumed during a paper grade change are technically wasted. The transitional production cost $C_t(i,j)$ during $j \mapsto k$ grade change period can be expressed as follows,

$$C_t(i,j) = \int_0^{\mu(j,k)} \Big(E(t,i,j) + R(t,i,j)\Big)dt. \tag{9}$$

Objective Function: As for the product scheduling problem, it is desired to find an optimal policy (schedule) for each order i so that the costs of such time allocation is minimised. Strictly speaking, the scheduling vector (2) needs to be optimised so that the overall costs imposed by the choice of production order is minimised. This statement can be formulated as follows,

$$J = \min_{\Pi} \sum_{i=1}^{m} \Big(C_p(i,j) + C_t(i,j)\Big) \times \Pi(i)^T, \tag{10}$$
$$\text{subject to } (3) - (6),$$

where, C_p and C_t are defined in (8) and (9). In the next section, the proposed solution to the optimisation problem above is presented.

4. APPROACH: MIXED-INTEGER PROGRAMMING

Considering the constraints (3)-(6), the optimisation problem introduced in section 3 is a binary integer programming problem with a quadratic constraint. It has been shown that quadratic inequality constraints in an LP problem can be formulated as Semi Definite Programming (SDP) problems (Vandenberghe and Boyd (1996)). In general, when the polynomial cost function and/or constraints become quadratic, the resulting optimisation problem becomes very hard and in some cases non-convex. Therefore, it becomes important to have easily computable lower bounds on the optimal value. The product scheduling problem introduced in (10) has two essential differences with standard quadratically constrained SDP problem. Firstly, the decision variable in scheduling problem is binary which makes it a binary integer programming problem. Secondly, unlike the standard quadratically constrained SDP problem, the scheduling problem includes quadratic 'equality' constraints. As such, the optimisation problem encountered in this paper can be considered as an SDP-based binary-integer programming problem. Here a branch and bound algorithm has been adopted to solve the binary integer programming problem introduced above.

SDP-based binary-integer programming: The general form of the SDP with quadratic cost and L quadratic constraints is expressed as follows,

$$J = \min_x f_0(x) \tag{11}$$

$$\text{subject to } f_p(x) \leq 0, \; p = 0, 1, \cdots, L, \tag{12}$$

where $f_p(x) = x^T A_p x + 2b_i^T x + c_i$ and matrix A_i can be indefinite which makes (11) a very hard non-convex optimisation problem. This problem can be transformed to the following dual form as shown in (Vandenberghe and Boyd (1996)),

$$J = \min_x \; \text{Trace} X A_0 + 2b_0^T x + c_0, \tag{13}$$

$$\text{subject to } \; \text{Trace} X A_p + 2b_p^T x + c_p, \tag{14}$$

$$\begin{bmatrix} X & x \\ x^T & I \end{bmatrix} \geq 0, \tag{15}$$

where decision variables are $X = X^T \in R^{L \times L}$ and $x \in R^L$. The scheduling optimisation objective (10) is in the form (11) with the following simplifications,

$$A_0 = 0, \; A_1 = I; \; c_0 = c_1 = 0; \; b_1 = 0 \tag{16}$$

$$b_0 = \frac{1}{2} \sum_{i=1}^{m} \Big(C_p(i,j) + C_t(i,j) \Big) \tag{17}$$

Also, the constraint (6) can equivalently be written as the following two inequality constraints,

$$\Pi(i) \times \Pi^T(j) \leq 0 \tag{18}$$

$$\Pi(i) \times \Pi^T(j) \geq 0 \tag{19}$$

Therefore, SDP problem (10) can be considered as the following binary integer programming problem,

$$J = \min_\Pi \sum_{i=1}^{m} \Big(C_p(i,j) + C_t(i,j) \Big) \times \Pi(i) \tag{20}$$

$$\text{subject to } \begin{bmatrix} 0 & \Pi(i) \\ \Pi(i)^T & I \end{bmatrix} \geq 0, \tag{21}$$

$$\text{and} \qquad (3), (4), (6). \tag{22}$$

The cost function above along with the constraints (3) , (5) and (6) will result in lower bound for the product scheduling problem. In order to solve the resulting binary integer programming problem, a branch and bound algorithm is employed where (3) is relaxed to a weaker constraint $\pi(i,j) \in [0,1]$, forming an SDP-relaxation problem. Then the lower bound of the binary integer problem is found by solving this SDP-relaxation problem. Two branches, i.e. $\pi_r = 0$ and $\pi_r = 1$ are generated (r= iterations of the binary integer programming) if the feasible solution to SDP problem (π_r) is not binary. If a new feasible sub-division (node) results in an integer point with lower objective value than that of the current integer point, the best feasible binary point is updated and the node search strategy moves to the next node. Obviously, if a node is infeasible, the whole branch containing the parent nodes is eliminated. In the next section, a simulation study demonstrating the effectiveness of proposed method is presented.

5. SIMULATION RESULTS

In order to demonstrate the effectiveness of the proposed product scheduling technique, two examples from a UK-based board machine are proposed in this section.

5.1 Example 1: Scheduling Single Orders

Let us consider a single order placed during a typical day of operation. It is assumed that the energy and raw material prices vary randomly but known a priori. This is a reasonable assumption as the energy required for steam generation is usually bough prior to product scheduling. During the day of study, five different paper grades (135, 150, 170, 171 and 200 *gsm*) were planned for production (i.e. $n = 5$), no paper breaks were reported, and process data were collected by sampling time $Ts = 30$ *sec*. There are often a set of nominal machine settings and raw material usage set-points related to different grades of paper. As for the studied mill, these settings as well as customer requirements are summarised in Table 1; (RA: Retention Aid). In addition, suppose that the delivery time

Table 1. Machine settings for different grades

Grade (j) (gsm)	$O(j)$ ($tonnes$)	$P(j)$ (t/hr)	RA1 usage (l/min)	RA2 usage (l/min)
135	44.5	26.7	38.4	28.7
150	108.2	28.6	42.7	31.0
170	165.7	28.9	43.0	31.1
171	181.7	28.9	43.7	31.7
200	170.7	26.3	45.0	32.5

set for each of the above mentioned grades are,

$$T_d = [2, 7.5, 15.7, 21.1, 18.4]^T$$

hours. Each set of above settings are usually identified by grade codes summarising all required machine settings and raw material requirements. For proprietary reasons, the mill's commercial product names are replaced by numbers 38, 42, 59, 45 and 44 in the order of appearance in Table 1. Fig. 3 shows the actual production schedule chosen in the mill during the day of study. As discussed in section 2 (and

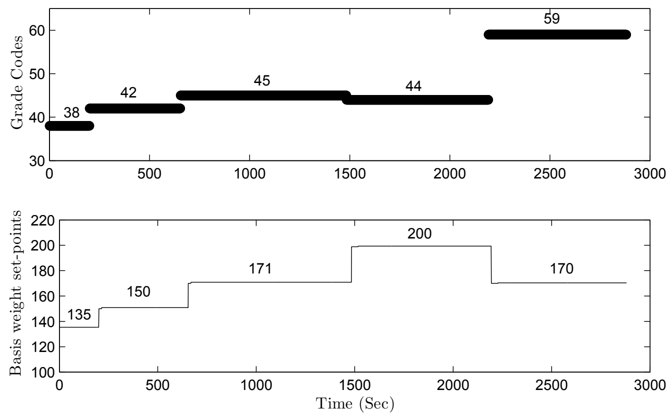

Fig. 3. Grade codes (top) and basis weight set-points (bottom) scheduled by the mill

shown in Fig. 3), the common practice of grade change is open loop ramping of the basis weight. In this section, a scheduling problem is solved considering the effect of energy and raw material costs during both steady state and grade transition modes. Although the main sources of energy in practice are steam, electricity and heated air; only steam is considered for optimal product scheduling in this paper to simplify the analysis. The energy and raw material prices are considered normally distributed random variables with standard deviation 20% and 2% of the nominal unit prices, respectively. Fig. 4 shows the variations in the unit prices of energy and raw materials for each grade. To solve the optimisation problem introduced

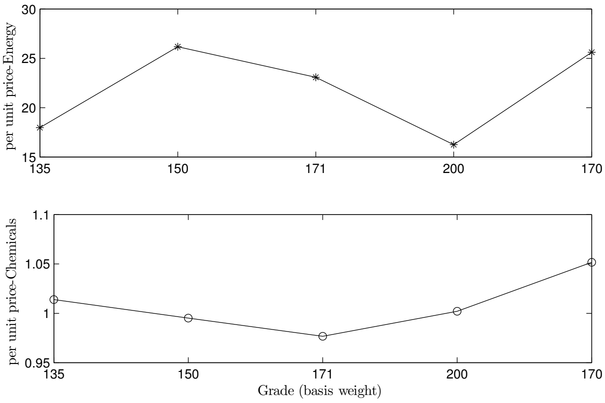

Fig. 4. Variations in per unit energy/material costs

in (20), a version of MATLAB's `bintprog` function is employed which is an LP-based branch-and-bound algorithm for solving binary integer programming problems. The branching strategy chosen in this paper is the maximum integer infeasibility which adds π=0 and π=1 to the binary solution tree when the value of variable π is close to 0.5. The constraints form the nodes of the binary map which is searched by the algorithm for the best lower bounds. As for the current example, 5 feasible nodes were found and the algorithm took total calculation time of 0.445 *sec* to find the optimum policy for product scheduling which is shown in Fig. 5. As the figure shows, the optimal policy incorpo-

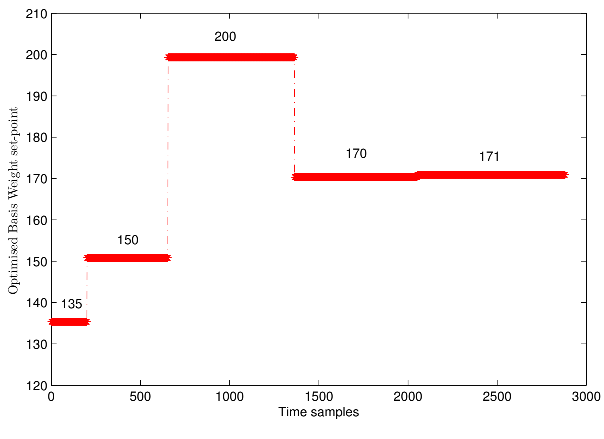

Fig. 5. Optimal schedule with varying production costs

rates producing the 170 and 171 grades after the heavier grade 200 *gsm*. This suggested change in the production schedule can be better understood by looking at steam consumption plot from the original schedule (that of Fig. 3) which is shown in Fig. 6 below. Comparing Fig. 4 and Fig. 6 against the obtained schedule, it can be seen that if the original schedule was chosen, at the peak of the energy cost, grade 171 (which has the greatest level of energy use among the last three grades) would be manufactured which contributed to less energy efficiency.Let us now consider a wider range of products and orders in the production plan.

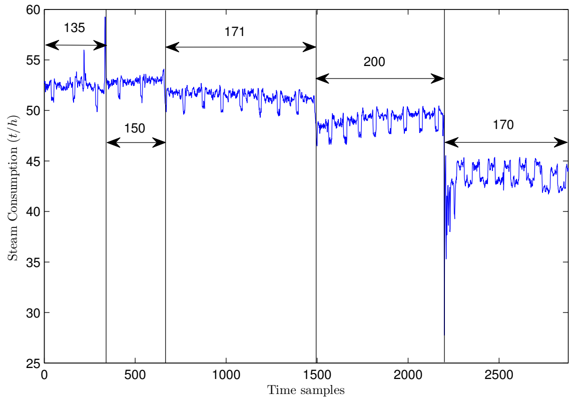

Fig. 6. Actual steam consumption for each grade

5.2 Example 2: Scheduling Multiple Orders

In this example, 4 orders each consisting of 4 grades (i.e 16 total paper grades) are needed to be scheduled for 8 days of operation. The sampling time for this case is 10 *sec* and the machine settings as well as customer requirements are summarised in Table 2. Similar to example 1, the

Table 2. Machine settings for different orders

Order1 grades (*gsm*)	140	120	112	100
Amount ($O(j)$, *tonnes*)	531.44	275.07	222.73	309.81
Delivery Time (T_d, *hr*)	153	177	209	223
Order2 grades (*gsm*)	64	99	74	92
Amount ($O(j)$, *tonnes*)	198.95	199.73	78.01	345.59
Delivery Time (T_d, *hr*)	64	99	74	92
Order3 grades (*gsm*)	200	220	170	150
Amount ($O(j)$, *tonnes*)	103.95	147.08	255.36	293.83
Delivery Time (T_d, *hr*)	96	101	104	116
Order4 grades (*gsm*)	135	115	105	140
Amount ($O(j)$, *tonnes*)	228.71	505.32	548.68	362.25
Delivery Time (T_d, *hr*)	125	145	170	183

common practice 'ramping' has been chosen at the mill for scheduling the production. Fig. 7 shows the grades codes as well as the schedule actually set in the mill. Furthermore,

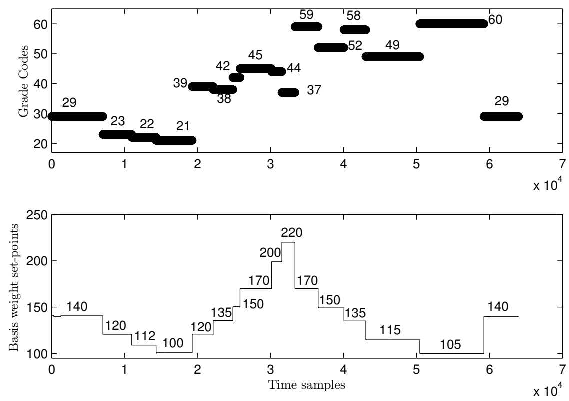

Fig. 7. The grade codes (top) and basis weights (bottom) scheduled for the machine

the variations of the energy and raw material costs is considered the same as that of example 1. That is, normally distributed random variables with standard deviation 20%

and 2% of the nominal unit prices. The variation of these costs per each grade code are shown in Fig. 8. Applying the

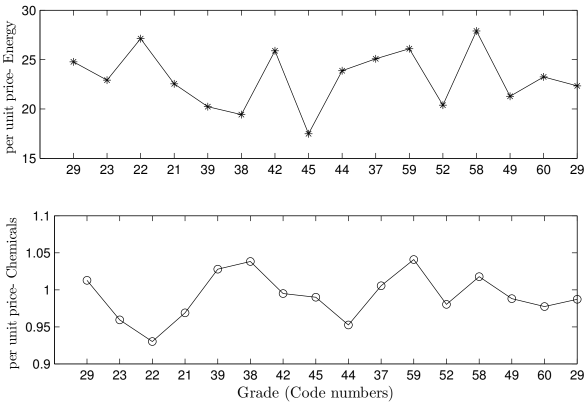

Fig. 8. Trend of cost variations

introduced binary integer programming algorithm with the same settings at those of example 1, the optimal schedule is determined by using 30 nodes and after 1.548 *sec*. Fig. 9 shows the obtained optimal schedule. Similar to

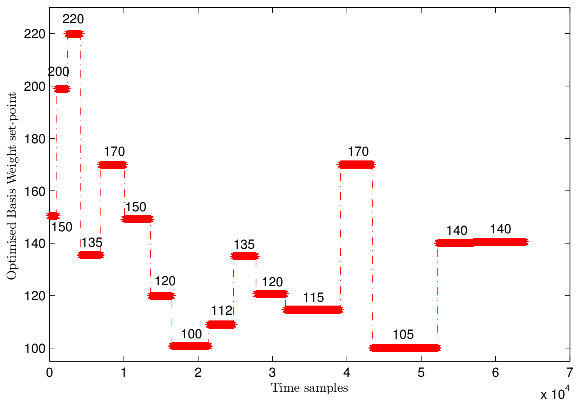

Fig. 9. Optimal schedule for multiple orders

example 1, scheduling is done so that the most cost efficient use of energy and raw material is realised considering the delivery deadlines. The suggested optimised schedule could result in £74,000 of total savings during 8 days which is equivalent to 0.06% saving in total production costs. It must be noted that without considering variations in the energy and raw material costs, the algorithm has to solve a makespan minimisation problem taking the delivery deadlines and the risk of paper breaks into account. Therefore, without considering the paper break constraints (see section 3), the proposed algorithm is ill-posed if no energy/raw material cost variations happen.The risks of paper breaks will be addressed in the future works.

6. CONCLUSIONS

A product scheduling algorithm to reduce the cost of thermal energy and raw material consumption in papermaking industry was proposed in this paper by taking both steady-state and grade transition costs into account. The optimal scheduling in single machines/multiple product cases has been considered in the literature for reducing product delivery time. In this paper, however, the problem is solved from an energy and production efficiency viewpoint. In such consideration, the energy consumption figures during both processing times and grade transition period have been considered. The optimisation problem associated with the production scheduling is then formulated as a quadratically constrained binary integer programming problem. The technique is tested on the operational data of a commercial board machine in the UK. It has been found that when energy prices vary (which is usually the case in papermaking), considering such variations in product scheduling can improve energy consumption, and consequently, increase profitability of the papermaking practice. Future work would involve modelling the paper breaks risk as new constraints and nonlinear modelling of the machine dynamics during the grade changes.

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