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Point-Line Collinearity Graphs of two Sporadic Minimal Parabolic Geometries

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Abstract

The disc structure of the point-line collinearity graph for the rank two minimal parabolic geometries of the Thompson and Harada-Norton simple groups are investigated. Additionally details of the sub-orbits of these two groups in their conjugation action upon an involution conjugacy class is given.

1 Introduction

This paper reports the outcome of calculations carried out on the point-line collinearity graphs for two sporadic minimal parabolic geometries. Both of these geometries have rank two and are of characteristic two. One of these geometries is associated with Th, Thompson's simple group and the other with HN, the Harada-Norton simple group. Apart from their intrinsic interest, the principal motivation for these investigations is that each of these graphs occur as full subgraphs of the Monster graph – see [6], [7], [8]. As a consequence the data assembled here will prove extremely valuable in unpicking some of the intricacies of the Monster graph, a graph associated with the Monster simple group which has approximately 5×10^{27} vertices.

The geometries Γ we study here were first described in [5] in terms of 2-minimal subgroups. So $\Gamma = \Gamma_0 \cup \Gamma_1$ where Γ_0 is the set of points of Γ and Γ_1 is the set of lines, with an incidence relation $*$. The point-line collinearity graph, $\mathcal{G}$, of Γ has $V(\mathcal{G}) = \Gamma_0$ as its vertex set with $x, y \in \Gamma_0$ adjacent provided $x \neq y$ and for some $\ell \in \Gamma_1$, $x * \ell$ and $y * \ell$. The usual graph theoretic distance on $\mathcal{G}$ will be denoted by $d(\ ,\ )$, and for $i \in \mathbb{N} \cup \{0\}$ and $x \in \Gamma_0$ define

$$\Delta_i(x) = \{y \in \Gamma_0 \mid d(x, y) = i\}.$$

We refer to $\Delta_i(x)$ as the i^{th} disc of x. Clearly $\Delta_i(x)$ is a union of certain G_x-orbits of Γ_0. It is these discs we examine here. For more background details on these and related geometries see [2].

Our first result concerns the point-line collinearity graph associated with Th – this graph has $976,841,775$ vertices.

Theorem 1. *Suppose that $G \cong Th$ and $\mathcal{G}$ is the point-line collinearity graph of the characteristic 2 minimal parabolic geometry for G. Then $\mathcal{G}$ has diameter 5 and for $t \in V(\mathcal{G})$ we have*

(i) $|\Delta_1(t)| = 270$ *with $\Delta_1(t)$ a G_t-orbit;*

(ii) $|\Delta_2(t)| = 64800$, *$\Delta_2(t)$ consisting of two G_t-orbits;*

(iii) $|\Delta_3(t)| = 15060480$, *$\Delta_3(t)$ consisting of six G_t-orbits;*

(iv) $|\Delta_4(t)| = 858497006$, *$\Delta_4(t)$ consisting of twenty six G_t-orbits; and*

(v) $|\Delta_5(t)| = 103219200$, *$\Delta_5(t)$ consisting of two G_t-orbits.*

The second graph we look at has fewer vertices, namely $74,064,375$.

Theorem 2. *Suppose that $G \cong HN$ and $\mathcal{G}$ is the point-line collinearity graph of the characteristic 2 minimal parabolic geometry for G. Then $\mathcal{G}$ has diameter 5 and for $t \in V(\mathcal{G})$ we have*

(i) $|\Delta_1(t)| = 150$ *with $\Delta_1(t)$ a G_t-orbit;*

(ii) $|\Delta_2(t)| = 17760$, *$\Delta_2(t)$ consisting of three G_t-orbits;*

(iii) $|\Delta_3(t)| = 1638400$, *$\Delta_3(t)$ consisting of eight G_t-orbits;*

(iv) $|\Delta_4(t)| = 68721664$, *$\Delta_4(t)$ consisting of fifty five G_t-orbits; and*

(v) $|\Delta_5(t)| = 3686400$, *$\Delta_5(t)$ consisting of three G_t-orbits.*

The information summarized in Theorems 1 and 2 was obtained with the assistance of MAGMA[3] using matrix representatives for Th and HN supplied by [10]. For the description of groups and the names of conjugacy classes we follow the ATLAS [4]. If x is a point in either of these two geometries, then it is the case that $G_x = C_G(i_x)$ where i_x is an involution of G and $Z(G_x) = \langle i_x \rangle$. Thus we may identify Γ_0 with $X = i_x^G$. Under this translation we have that $\Delta_1(x)$ becomes $(X \cap O_2(C_G(i_x))) \setminus \{i_x\}$. It is in this arena that we uncover the structure the discs of the point-line collinearity graph. The sub-orbits of G in its conjugation action on X form an important part of our investigation and involves analyzing the sets X_C defined by

$$X_C = \{x \in X \mid tx \in C\},$$

for C a conjugacy class of G and t a fixed element of X. Clearly each non-empty set X_C is a union of certain $C_G(t)$-orbits. The sizes of these sets for each C can easily be determined from the complex character table of G. In Section 2 further details are given as to how our calculations were performed. Section 3 contains the collapsed adjacency matrices for these two collinearity graphs, from which Theorems 1 and 2 readily follow. Finally, in Section 4, suborbit representatives are listed for the action of Th and HN on the vertices of these two graphs. A computer file containing these representatives may be obtained from the first named author on request.

2 Determining the $C_G(t)$-orbits

2.1 $G \cong Th$

Much of the methodology for obtaining orbit representatives in this case follows the work done in [9].

Let a, b be the standard generators for G provided in [10], so that a is in class $2A$, b is in class $3A$, ab has order 19, and $\langle a, b\rangle = G$. We set $t = a, X = 2A$, then generators for the maximal subgroup $C_G(t)$ of shape $2_+^{1+8}.A_9$ can be obtained from the straight line program provided in [10]. The representation used is as 248×248 matrices over $GF(2)$.

We set $Q = O_2(C_G(t))$. So $Q \cong 2_+^{1+8} \trianglelefteq C_G(t)$. Generators for Q can easily be obtained by taking random elements of $C_G(t)$ until we find elements having order 36 and taking their 9^{th} powers, enough of which along with t will generate Q.

Direct computation in $C_G(t)$ is precluded by its size and the large degree of the representation. However, since Q is small and a normal subgroup, we may construct explicitly the conjugation action of $C_G(t)$ on Q, giving us (after relabelling the elements of Q as $\{1,\ldots,512\}$) a homomorphism $\varphi : C_G(t) \to M \leq \mathrm{Sym}(512)$ with $M \cong C_G(t)/\langle t\rangle$. With this set up in place, we describe how we go about finding representatives for the $C_G(t)$-orbits of X.

From information gleaned from the character table, we know that X consists of thirty-eight $C_G(t)$-orbits across 29 non-empty sets X_C of known size. So we know that at least twenty of the non-empty sets X_C each consist of a single $C_G(t)$-orbit, and at most nine sets X_C split into two or more $C_G(t)$-orbits. Our strategy is to take a random conjugate x of t, determine the class of $z = tx$ and so which X_C contains the orbit, and where possible the size of $C_{C_G(t)}(x)$. This then yields the size of the orbit and eventually helps in finding representatives for all thirty-eight $C_G(t)$-orbits. The class of z is generally immediately apparent from its order and the dimension of its fixed space on the 248-dimensional $GF(2)$-module V, so we focus on the second task: determining the size of $C_{C_G(t)}(x)$.

Since $C_{C_G(t)}(x) \leq C_G(z)$, if we can compute $C_G(z)$ and if it is sufficiently small, we can then find $C_{C_G(t)}(x)$ directly in this group. In particular, where z is in classes $18B, 19A, 20A, 21A, 27A, 28A, 36A$, we find $C_G(z) = \langle z\rangle$ and so this is easily done. When z is in a class of order 3 we can compute $C_G(z)$ using [1] (and when z has order $3n$ for some $n > 1$ we can compute its centralizer inside $C_G(z^n)$ which we find by the same method). Finally, when z is in class $C = 13A$, we observe that $C_G(z)$ has order 39, so $C_{C_G(t)}(x)$ must have order 1 or 3 (since 13 does not divide $|C_G(t)|$). But $|X_C| = 30965760 = |C_G(t)|/3$, and this forces $|C_{C_G(t)}(x)|$ to have size 3, so we know X_{13A} is a single $C_G(t)$-orbit.

Now suppose C is a class of elements of even order $2m$. Here we have the advantage that z^m is an involution commuting with t and x, so $z^m \in C_{C_G(t)}(x)$. Recall that we have $C_G(t)/\langle t\rangle \cong M \leq \mathrm{Sym}(512)$ corresponding to the conjugation action of $C_G(t)$ on Q. We have that $C_{C_G(t)}(x) = C_{C_G(t)}(z) \leq C_{C_G(t)}(z^m)$, and crucially, $z^m \in C_G(t)$. Now we have $\varphi : C_G(t) \to M$ explicitly, so we can calculate $C_M(\varphi(z^m))$, take its inverse image, and if it is sufficiently small, calculate $C_{C_G(t)}(x)$ in this. Further, since Q is small, we can compute $C_Q(x) \trianglelefteq C_{C_G(t)}(x)$. Since the points of M correspond to the elements of Q, we can form S_x, the stabilizer in M of the subset of $\{1, \ldots, 512\}$ corresponding to $C_Q(x)$, and $C_{C_G(t)}(x)$ must lie in its inverse image. In all cases where z has even order, we find that $S_x \cap C_M(\phi(z^m))$ is sufficiently small to be able to compute $C_{C_G(t)}(x)$ in its inverse image.

These calculations are sufficient to uncover nine sets X_C that each split into two $C_G(t)$-orbits, so we conclude that the remaining unanalysed X_C (namely X_{5A}, X_{7A}) are single orbits.

2.2 $G \cong HN$

Finding representatives for the $C_G(t)$-orbits when $G \cong HN, t \in X = 2B$ poses a different set of problems to those encountered in the Thompson group. In particular, here we have that $C_G(t)$ is relatively small, containing just 3,686,400 elements. This means computation inside $C_G(t)$ can be carried out directly in the matrix group, as can $C_G(t)$-conjugacy testing, making the discovery of new orbits and their sizes much easier. (We use the representation of G as 132×132 matrices over $GF(4)$ from [10].) On the other hand, there are more $C_G(t)$-orbits to find: 70 orbits across 47 non-empty X_C, and many classes of G cannot be identified so easily by the dimensions of their fixed spaces on the 132-dimensional module V. Therefore our strategy is as follows.

We take a random conjugate x of t, and compute four easily-determined orbit invariants: the order of $z = tx$, the dimension of the fixed space of z, the value $d_x = \dim(C_V(t) \cap C_V(x))$ and the order of $C_{C_G(t)}(x)$ (and hence the size of the orbit). From the sets X_C we know if we have found all the orbits for a particular order of z and if this is the case the representative is discarded. Otherwise, we test whether the representative is $C_G(t)$-conjugate to any representative sharing its invariants that we have already found and if not add it to our list. (This conjugacy testing is initially carried out using Magma's `IsConjugate` command, but as this has been found to be fallible in this situation the results are confirmed by other methods.) We leave determining the exact class in which z lies until after all the representatives have been found.

When a representative x for a new orbit is found we also form its 'powers' tz^n for all values of n dividing the order of $z = tx$, and check these as well. Suppose we have an element $x \in X_C$ for some class C of G. Further suppose that for $z \in C$, $(z^n)^G = D$ (C 'powers down' to D). Then we have $x' = tz^n =$

$xtx \dots tx \in X$. Clearly $tx' \in D$ and so $x' \in X_D$. Further, all elements from a particular $C_G(t)$-orbit in X_C power down in this fashion to the same orbit in X_D. Since the smaller $C_G(t)$-orbits tend to lie in X_C for classes C having elements of small orders, this powering down strategy can prove very helpful in unearthing these elusive orbits. We note that forming an x' in this manner also furnishes us with a conjugating element h such that $t^h = x'$ in terms of t and the element g conjugating t to x, reducing the number and length of words needed to obtain representatives for every orbit. We observe that we can define representatives for all seventy $C_G(t)$-orbits using only 34 words (see Section 4.2).

3 Collapsed Adjacency Matrices

Once we have a list of representatives and conjugating elements for the $C_G(t)$-orbits, we determine the collapsed adjacency matrix for the point-line collinearity graphs. Recall that each disc of the graph is a union of G_t-orbits (which we view as $C_G(t)$-orbits on X). The collapsed adjacency matrix has one such orbit associated with each row and with each column. Then the $(i, j)^{\text{th}}$ entry of the matrix gives the number of edges going from a particular arbitrary point in the associated orbit $\mathcal{O}_i$ to any point in the associated orbit $\mathcal{O}_j$.

Our first task in constructing this matrix is to create an explicit listing of the elements of $\Delta_1(t) = X \cap O_2(C_G(t))$. This is easily done in both cases. (Note that for $G \cong Th$, the group $O_2(C_G(t))$ is just the group Q described in Section 2.1.) Then for each $C_G(t)$-orbit representative x and conjugating element g (so $t^g = x$, see Section 4), we compute $\Delta_1(x) = \Delta_1(t)^g$. We determine in which orbit each element of $\Delta_1(x)$ lies, and so build a 'neighbourhood profile' for every orbit. From this data it is simple to locate which disc a given orbit is in, and to create the collapsed adjacency matrix.

Orbits in the matrix are ordered first in terms of increasing distance from t and then in terms of increasing size.

3.1 $G \cong Th$

The matrix is broken into four tables across the following four pages

	Δ_0^1	Δ_1^1	Δ_2^1	Δ_2^2	Δ_3^1	Δ_3^2	Δ_3^3	Δ_3^4	Δ_3^5	Δ_3^6
Δ_0^1	0	270	0	0	0	0	0	0	0	0
Δ_1^1	1	29	112	128	0	0	0	0	0	0
Δ_2^1	0	1	13	16	0	16	96	0	0	128
Δ_2^2	0	1	14	15	16	0	0	112	112	0
Δ_3^1	0	0	0	9	9	0	0	126	126	0
Δ_3^2	0	0	1	0	0	1	12	0	0	16
Δ_3^3	0	0	1	0	0	2	11	0	0	16
Δ_3^4	0	0	0	1	2	0	0	21	14	8
Δ_3^5	0	0	0	1	2	0	0	14	13	0
Δ_3^6	0	0	1	0	0	2	12	8	0	23
Δ_4^1	0	0	0	0	0	9	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	27
Δ_4^3	0	0	0	0	0	1	0	0	8	0
Δ_4^4	0	0	0	0	0	0	5	0	0	0
Δ_4^5	0	0	0	0	0	0	9	0	0	0
Δ_4^6	0	0	0	0	0	0	0	2	2	1
Δ_4^7	0	0	0	0	0	0	0	4	0	5
Δ_4^8	0	0	0	0	0	0	0	0	2	3
Δ_4^9	0	0	0	0	0	0	0	6	0	3
Δ_4^{10}	0	0	0	0	0	2	0	2	0	1
Δ_4^{11}	0	0	0	0	0	0	0	0	2	3
Δ_4^{12}	0	0	0	0	0	0	1	0	0	0
Δ_4^{13}	0	0	0	0	0	1	0	6	2	3
Δ_4^{14}	0	0	0	0	0	0	3	0	2	0
Δ_4^{15}	0	0	0	0	0	0	0	2	2	1
Δ_4^{16}	0	0	0	0	0	0	1	0	0	0
Δ_4^{17}	0	0	0	0	0	0	0	0	2	3
Δ_4^{18}	0	0	0	0	0	0	2	2	0	1
Δ_4^{19}	0	0	0	0	0	1	0	0	2	0
Δ_4^{20}	0	0	0	0	0	0	1	0	2	0
Δ_4^{21}	0	0	0	0	0	0	2	2	0	1
Δ_4^{22}	0	0	0	0	0	0	0	0	2	3
Δ_4^{23}	0	0	0	0	0	0	1	0	0	0
Δ_4^{24}	0	0	0	0	0	0	1	0	0	0
Δ_4^{25}	0	0	0	0	0	0	1	0	2	0
Δ_4^{26}	0	0	0	0	0	0	0	2	0	1
Δ_5^1	0	0	0	0	0	0	0	0	0	0
Δ_5^2	0	0	0	0	0	0	0	0	0	0

	Δ_4^1	Δ_4^2	Δ_4^3	Δ_4^4	Δ_4^5	Δ_4^6	Δ_4^7	Δ_4^8	Δ_4^9
Δ_0^1	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0
Δ_3^2	8	0	8	0	0	0	0	0	0
Δ_3^3	0	0	0	8	16	0	0	0	0
Δ_3^4	0	0	0	0	0	8	16	0	24
Δ_3^5	0	0	8	0	0	8	0	8	0
Δ_3^6	0	4	0	0	0	4	20	12	12
Δ_4^1	0	0	9	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	27	27	0
Δ_4^3	1	0	0	0	0	8	0	8	0
Δ_4^4	0	0	0	0	10	0	0	0	0
Δ_4^5	0	0	0	9	9	0	0	0	0
Δ_4^6	0	0	2	0	0	0	2	8	3
Δ_4^7	0	1	0	0	0	2	4	3	12
Δ_4^8	0	1	2	0	0	8	3	2	0
Δ_4^9	0	0	0	0	0	3	12	0	6
Δ_4^{10}	2	0	2	0	0	7	2	6	3
Δ_4^{11}	0	1	2	0	0	2	9	5	6
Δ_4^{12}	0	0	0	1	2	4	4	4	4
Δ_4^{13}	1	0	3	0	0	5	9	2	12
Δ_4^{14}	0	0	2	5	6	4	0	4	0
Δ_4^{15}	0	0	2	0	0	5	6	4	7
Δ_4^{16}	0	0	0	1	2	8	0	8	0
Δ_4^{17}	0	1	2	2	0	6	3	9	0
Δ_4^{18}	0	0	0	4	4	3	4	2	5
Δ_4^{19}	1	0	3	0	0	6	2	6	2
Δ_4^{20}	0	0	2	1	2	2	6	2	6
Δ_4^{21}	0	0	0	2	4	3	6	2	7
Δ_4^{22}	0	1	2	0	0	6	5	9	2
Δ_4^{23}	0	0	0	1	2	4	4	4	4
Δ_4^{24}	0	0	0	1	2	4	4	4	4
Δ_4^{25}	0	0	2	1	2	4	4	4	4
Δ_4^{26}	0	0	0	0	0	5	6	4	7
Δ_5^1	0	0	0	0	0	0	9	0	9
Δ_5^2	0	0	0	3	0	3	3	3	3

	Δ_4^{10}	Δ_4^{11}	Δ_4^{12}	Δ_4^{13}	Δ_4^{14}	Δ_4^{15}	Δ_4^{16}	Δ_4^{17}	Δ_4^{18}	Δ_4^{19}
Δ_0^1	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0
Δ_3^2	64	0	0	64	0	0	0	0	0	96
Δ_3^3	0	0	8	0	48	0	16	0	32	0
Δ_3^4	8	0	0	48	0	24	0	0	24	0
Δ_3^5	0	8	0	16	24	24	0	24	0	24
Δ_3^6	4	12	0	24	0	12	0	36	12	0
Δ_4^1	72	0	0	72	0	0	0	0	0	108
Δ_4^2	0	27	0	0	0	0	0	81	0	0
Δ_4^3	8	8	0	24	24	24	0	24	0	36
Δ_4^4	0	0	5	0	50	0	10	20	40	0
Δ_4^5	0	0	9	0	54	0	18	0	36	0
Δ_4^6	7	2	6	10	12	15	24	18	9	18
Δ_4^7	2	9	6	18	0	18	0	9	12	6
Δ_4^8	6	5	6	4	12	12	24	27	6	18
Δ_4^9	3	6	6	24	0	21	0	0	15	6
Δ_4^{10}	14	0	6	22	6	9	24	12	9	36
Δ_4^{11}	0	2	6	10	6	18	0	15	6	12
Δ_4^{12}	4	4	0	4	10	12	18	8	12	12
Δ_4^{13}	11	5	3	24	6	21	0	6	12	21
Δ_4^{14}	2	2	5	4	27	8	14	20	24	10
Δ_4^{15}	3	6	6	14	8	10	8	10	9	14
Δ_4^{16}	8	0	9	0	14	8	25	16	12	16
Δ_4^{17}	4	5	4	4	20	10	16	24	14	14
Δ_4^{18}	3	2	6	8	24	9	12	14	16	6
Δ_4^{19}	12	4	6	14	10	14	16	14	6	19
Δ_4^{20}	0	8	7	10	12	18	2	6	10	12
Δ_4^{21}	3	4	8	10	14	13	12	4	17	8
Δ_4^{22}	4	7	6	6	10	14	16	23	6	16
Δ_4^{23}	4	4	9	4	10	12	18	8	12	12
Δ_4^{24}	4	4	9	4	10	12	18	8	12	12
Δ_4^{25}	2	6	7	8	14	16	10	10	10	14
Δ_4^{26}	5	4	8	10	4	15	16	8	11	12
Δ_5^1	0	9	9	9	0	18	0	0	9	9
Δ_5^2	3	3	6	3	18	9	12	21	21	9

	Δ_4^{20}	Δ_4^{21}	Δ_4^{22}	Δ_4^{23}	Δ_4^{24}	Δ_4^{25}	Δ_4^{26}	Δ_5^{1}	Δ_5^{2}
Δ_0^1	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0
Δ_3^3	16	32	0	16	16	32	0	0	0
Δ_3^4	0	24	0	0	0	0	48	0	0
Δ_3^5	24	0	24	0	0	48	0	0	0
Δ_3^6	0	12	36	0	0	0	24	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	81	0	0	0	0	0	0
Δ_4^3	24	0	24	0	0	48	0	0	0
Δ_4^4	10	20	0	10	10	20	0	0	60
Δ_4^5	18	36	0	18	18	36	0	0	0
Δ_4^6	6	9	18	12	12	24	30	0	18
Δ_4^7	18	18	15	12	12	24	36	6	18
Δ_4^8	6	6	27	12	12	24	24	0	18
Δ_4^9	18	21	6	12	12	24	42	6	18
Δ_4^{10}	0	9	12	12	12	12	30	0	18
Δ_4^{11}	24	12	21	12	12	36	24	6	18
Δ_4^{12}	14	16	12	18	18	28	32	4	24
Δ_4^{13}	15	15	9	6	6	24	30	3	9
Δ_4^{14}	12	14	10	10	10	28	8	0	36
Δ_4^{15}	18	13	14	12	12	32	30	4	18
Δ_4^{16}	2	12	16	18	18	20	32	0	24
Δ_4^{17}	6	4	23	8	8	20	16	0	42
Δ_4^{18}	10	17	6	12	12	20	22	2	42
Δ_4^{19}	12	8	16	12	12	28	24	2	18
Δ_4^{20}	17	16	12	14	14	40	24	6	18
Δ_4^{21}	16	12	8	16	16	28	30	4	18
Δ_4^{22}	12	8	16	12	12	28	24	2	18
Δ_4^{23}	14	16	12	9	18	28	32	4	24
Δ_4^{24}	14	16	12	18	9	28	32	4	24
Δ_4^{25}	20	14	14	14	14	27	24	4	18
Δ_4^{26}	12	15	12	16	16	24	29	4	24
Δ_5^1	27	18	9	18	18	36	36	0	27
Δ_5^2	9	9	9	12	12	18	24	3	54

3.2 $G \cong HN$

The graph is presented in twelve pieces scanning across the table in two rows.

	Δ_0^1	Δ_1^1	Δ_2^1	Δ_2^2	Δ_2^3	Δ_3^1	Δ_3^2	Δ_3^3	Δ_3^4	Δ_3^5	Δ_3^6	Δ_3^7
Δ_0^1	0	150	0	0	0	0	0	0	0	0	0	0
Δ_1^1	1	5	32	48	64	0	0	0	0	0	0	0
Δ_2^1	0	5	5	0	20	0	0	120	0	0	0	0
Δ_2^2	0	1	0	5	0	0	16	32	16	16	0	0
Δ_2^3	0	1	2	0	11	16	0	24	0	0	48	48
Δ_3^1	0	0	0	0	6	6	0	9	0	0	9	0
Δ_3^2	0	0	0	1	0	0	1	0	2	2	0	0
Δ_3^3	0	0	1	2	2	2	0	9	0	0	14	0
Δ_3^4	0	0	0	1	0	0	2	0	1	2	0	0
Δ_3^5	0	0	0	1	0	0	2	0	2	1	0	0
Δ_3^6	0	0	0	0	2	1	0	7	0	0	18	8
Δ_3^7	0	0	0	0	1	0	0	0	0	0	4	9
Δ_3^8	0	0	0	1	0	0	0	2	0	0	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	0	30	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	9	0	0	0	0	0
Δ_4^4	0	0	0	0	0	0	6	3	0	0	0	0
Δ_4^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^6	0	0	0	0	0	0	0	5	0	0	0	0
Δ_4^7	0	0	0	0	0	0	0	5	0	0	0	0
Δ_4^8	0	0	0	0	0	0	0	0	0	5	0	0
Δ_4^9	0	0	0	0	0	0	0	0	5	0	0	0
Δ_4^{10}	0	0	0	0	0	5	0	5	0	0	5	10
Δ_4^{11}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{12}	0	0	0	0	0	1	0	0	0	0	3	0
Δ_4^{13}	0	0	0	0	0	0	2	1	0	0	4	0
Δ_4^{14}	0	0	0	0	0	0	0	1	2	0	0	0
Δ_4^{15}	0	0	0	0	0	2	0	1	0	0	0	2
Δ_4^{16}	0	0	0	0	0	0	0	1	0	0	2	2
Δ_4^{17}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_4^{18}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{19}	0	0	0	0	0	0	1	0	0	0	0	0
Δ_4^{20}	0	0	0	0	0	0	0	1	0	0	2	2
Δ_4^{21}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{22}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{23}	0	0	0	0	0	0	0	1	0	2	2	0

	Δ_3^8	Δ_4^1	Δ_4^2	Δ_4^3	Δ_4^4	Δ_4^5	Δ_4^6	Δ_4^7	Δ_4^8	Δ_4^9	Δ_4^{10}	Δ_4^{11}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	64	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	36	0
Δ_3^2	0	0	0	8	8	0	0	0	0	0	0	0
Δ_3^3	8	0	0	0	4	0	8	8	0	0	8	0
Δ_3^4	0	0	0	0	0	0	0	0	0	8	0	0
Δ_3^5	0	0	0	0	0	0	0	0	8	0	0	0
Δ_3^6	0	2	0	0	0	0	0	0	0	0	4	0
Δ_3^7	0	0	0	0	0	0	0	0	0	0	4	0
Δ_3^8	15	0	2	0	2	2	0	0	0	0	0	2
Δ_4^1	0	0	0	0	20	0	0	0	0	0	0	0
Δ_4^2	25	0	0	0	0	25	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	6	2	0	0	0	0	0	0	0	0	0	0
Δ_4^5	5	0	5	0	0	0	0	0	0	0	0	0
Δ_4^6	0	0	0	0	0	0	5	0	0	0	0	0
Δ_4^7	0	0	0	0	0	0	0	5	0	0	0	0
Δ_4^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^9	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{10}	0	0	0	0	0	0	0	0	0	0	15	0
Δ_4^{11}	5	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{12}	6	1	0	0	1	0	0	0	0	0	0	0
Δ_4^{13}	6	0	0	0	3	0	0	0	0	0	0	0
Δ_4^{14}	2	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{15}	0	0	0	0	0	0	0	0	0	0	2	0
Δ_4^{16}	2	0	0	0	0	0	2	0	0	0	0	0
Δ_4^{17}	0	0	0	0	0	4	0	0	0	1	0	0
Δ_4^{18}	1	0	0	0	0	0	0	0	0	0	0	6
Δ_4^{19}	0	0	0	1	0	0	0	0	0	0	4	0
Δ_4^{20}	2	0	0	0	0	0	0	2	0	0	0	0
Δ_4^{21}	1	0	0	0	0	0	0	0	4	0	0	0
Δ_4^{22}	1	0	0	0	0	0	0	0	0	4	0	0
Δ_4^{23}	2	0	0	0	0	0	2	0	0	0	2	0

	Δ_4^{12}	Δ_4^{13}	Δ_4^{14}	Δ_4^{15}	Δ_4^{16}	Δ_4^{17}	Δ_4^{18}	Δ_4^{19}	Δ_4^{20}	Δ_4^{21}	Δ_4^{22}	Δ_4^{23}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	12	0	0	72	0	0	0	0	0	0	0	0
Δ_3^2	0	8	0	0	0	0	0	8	0	0	0	0
Δ_3^3	0	4	8	8	8	0	0	0	8	0	0	8
Δ_3^4	0	0	16	0	0	8	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	16
Δ_3^6	4	8	0	0	8	0	0	0	8	0	0	8
Δ_3^7	0	0	0	4	4	0	0	0	4	0	0	0
Δ_3^8	4	6	4	0	4	0	2	0	4	2	2	4
Δ_4^1	20	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	9	0	0	0	0
Δ_4^4	2	9	0	0	0	0	0	0	0	0	0	0
Δ_4^5	0	0	0	0	0	20	0	0	0	0	0	0
Δ_4^6	0	0	0	0	10	0	0	0	0	0	0	10
Δ_4^7	0	0	0	0	0	0	0	0	10	0	0	0
Δ_4^8	0	0	0	0	0	0	0	0	0	20	0	0
Δ_4^9	0	0	0	0	0	5	0	0	0	0	20	0
Δ_4^{10}	0	0	0	10	0	0	0	20	0	0	0	10
Δ_4^{11}	0	0	0	0	0	0	30	0	0	0	0	0
Δ_4^{12}	11	3	0	6	0	0	0	12	0	0	0	0
Δ_4^{13}	2	6	0	4	0	0	4	0	0	4	4	0
Δ_4^{14}	0	0	3	6	4	4	0	0	0	0	4	0
Δ_4^{15}	2	2	6	11	4	0	0	8	4	0	0	0
Δ_4^{16}	0	0	4	4	5	0	4	4	6	0	0	6
Δ_4^{17}	0	0	4	0	0	4	0	0	4	0	4	0
Δ_4^{18}	0	2	0	0	4	0	4	0	0	2	1	4
Δ_4^{19}	4	0	0	8	4	0	0	12	4	0	0	4
Δ_4^{20}	0	0	0	4	6	4	0	4	5	0	0	0
Δ_4^{21}	0	2	0	0	0	0	2	0	0	4	0	4
Δ_4^{22}	0	2	4	0	0	4	1	0	0	0	4	0
Δ_4^{23}	0	0	0	0	6	0	4	4	0	4	0	17

	Δ_4^{24}	Δ_4^{25}	Δ_4^{26}	Δ_4^{27}	Δ_4^{28}	Δ_4^{29}	Δ_4^{30}	Δ_4^{31}	Δ_4^{32}	Δ_4^{33}	Δ_4^{34}	Δ_4^{35}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	16	0	0	32	0	0	0
Δ_3^3	0	8	8	8	0	8	8	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	16	0	0	0	0	0
Δ_3^5	0	0	0	16	8	0	0	0	0	16	0	0
Δ_3^6	0	8	8	0	0	8	8	0	0	0	0	0
Δ_3^7	0	4	4	0	0	0	0	0	8	0	0	8
Δ_3^8	2	4	4	4	0	16	4	2	0	0	4	4
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	0	6	6	0	0	6	0	0	24	0	0	0
Δ_4^5	20	0	0	0	20	20	0	0	0	0	0	0
Δ_4^6	0	0	10	0	0	10	0	0	0	0	0	0
Δ_4^7	0	10	0	0	0	10	10	0	0	0	0	0
Δ_4^8	0	0	0	0	5	0	0	0	0	30	0	0
Δ_4^9	0	0	0	0	0	0	0	0	0	0	40	0
Δ_4^{10}	0	0	0	0	0	0	10	0	0	0	0	0
Δ_4^{11}	5	10	10	0	0	0	0	30	0	0	0	0
Δ_4^{12}	0	6	6	0	0	0	0	0	6	0	0	12
Δ_4^{13}	0	6	6	0	0	2	0	4	4	8	0	0
Δ_4^{14}	0	2	0	2	4	2	2	0	4	4	8	8
Δ_4^{15}	0	0	0	6	0	0	0	0	0	4	8	4
Δ_4^{16}	0	2	0	0	4	2	0	0	0	4	4	0
Δ_4^{17}	4	0	0	4	0	4	0	0	4	0	8	0
Δ_4^{18}	0	2	8	0	0	0	0	12	10	12	2	2
Δ_4^{19}	0	0	0	0	0	4	4	0	4	4	4	4
Δ_4^{20}	0	0	2	4	0	2	6	4	0	0	0	8
Δ_4^{21}	2	0	10	4	4	0	0	1	4	8	0	0
Δ_4^{22}	2	10	0	0	0	0	4	2	4	0	18	6
Δ_4^{23}	0	0	4	2	0	0	2	0	4	16	0	0

	Δ_4^{36}	Δ_4^{37}	Δ_4^{38}	Δ_4^{39}	Δ_4^{40}	Δ_4^{41}	Δ_4^{42}	Δ_4^{43}	Δ_4^{44}	Δ_4^{45}	Δ_4^{46}	Δ_4^{47}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	16	0	0	0	0	0	0	0	0	0	0	16
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	16	0	32	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	32	0	0	0
Δ_3^6	0	0	8	0	8	8	8	0	0	0	0	0
Δ_3^7	0	0	8	8	0	0	8	0	8	0	8	0
Δ_3^8	0	0	4	4	8	4	4	4	4	4	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	50	0	50	0	0
Δ_4^3	18	0	0	0	36	0	0	0	0	0	36	18
Δ_4^4	0	24	0	0	0	0	0	0	0	0	0	0
Δ_4^5	0	0	0	0	0	0	0	10	0	10	0	0
Δ_4^6	0	20	0	0	0	0	10	0	0	0	0	20
Δ_4^7	20	0	10	0	0	10	0	0	0	0	0	0
Δ_4^8	0	0	20	0	0	0	0	0	0	0	0	0
Δ_4^9	0	10	0	0	0	0	20	0	0	0	0	0
Δ_4^{10}	0	0	0	0	0	0	0	0	0	0	20	0
Δ_4^{11}	0	0	20	10	0	0	20	0	10	0	0	0
Δ_4^{12}	0	0	0	0	6	0	0	0	0	0	6	0
Δ_4^{13}	8	0	0	8	12	0	0	0	8	0	4	8
Δ_4^{14}	4	4	2	4	0	6	0	8	8	8	4	8
Δ_4^{15}	4	4	0	4	8	8	0	0	4	0	8	4
Δ_4^{16}	4	0	4	4	4	12	0	0	4	0	8	4
Δ_4^{17}	0	10	0	0	4	4	0	8	4	12	4	8
Δ_4^{18}	8	4	12	4	0	0	12	0	0	4	0	0
Δ_4^{19}	2	0	4	0	8	0	4	4	0	4	12	2
Δ_4^{20}	4	8	0	4	4	4	4	0	4	0	8	4
Δ_4^{21}	12	0	4	0	4	4	0	0	8	4	2	0
Δ_4^{22}	0	4	0	8	4	4	4	4	0	0	2	12
Δ_4^{23}	4	0	16	0	4	4	0	0	8	0	0	0

	Δ_4^{48}	Δ_4^{49}	Δ_4^{50}	Δ_4^{51}	Δ_4^{52}	Δ_4^{53}	Δ_4^{54}	Δ_4^{55}	Δ_5^{1}	Δ_5^{2}	Δ_5^{3}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	32	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	16	0	0	0	32	0	0	0	0
Δ_3^5	0	16	0	0	0	0	0	32	0	0	0
Δ_3^6	8	0	0	0	0	0	0	0	0	0	0
Δ_3^7	0	0	0	0	8	16	16	16	0	0	0
Δ_3^8	4	0	0	4	4	0	0	0	0	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	80	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	12	12	0
Δ_4^4	0	24	0	0	0	0	0	0	12	8	12
Δ_4^5	0	0	0	0	0	0	20	20	0	0	0
Δ_4^6	10	0	0	0	0	20	0	0	0	0	20
Δ_4^7	0	20	0	0	0	20	0	0	0	0	20
Δ_4^8	0	10	0	40	0	0	0	20	0	0	0
Δ_4^9	0	0	30	0	0	0	20	0	0	0	0
Δ_4^{10}	0	0	0	0	0	20	0	0	0	0	20
Δ_4^{11}	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{12}	0	0	0	0	12	0	12	12	6	4	18
Δ_4^{13}	0	0	8	0	0	8	8	8	0	0	0
Δ_4^{14}	4	0	0	0	0	12	12	0	0	0	4
Δ_4^{15}	8	4	4	8	4	4	0	0	0	4	4
Δ_4^{16}	4	8	0	0	8	4	0	16	0	8	0
Δ_4^{17}	0	0	6	4	4	16	4	8	4	0	4
Δ_4^{18}	4	4	0	0	0	0	0	16	0	4	6
Δ_4^{19}	0	0	4	4	4	4	4	4	0	4	16
Δ_4^{20}	12	0	4	4	0	4	16	0	0	8	0
Δ_4^{21}	4	4	0	18	6	8	0	16	2	0	4
Δ_4^{22}	4	0	8	0	0	8	16	0	2	0	4
Δ_4^{23}	0	4	0	4	8	4	4	8	0	4	4

	Δ_0^1	Δ_1^1	Δ_2^1	Δ_2^2	Δ_2^3	Δ_3^1	Δ_3^2	Δ_3^3	Δ_3^4	Δ_3^5	Δ_3^6	Δ_3^7
Δ_4^{24}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{25}	0	0	0	0	0	0	0	1	0	0	2	2
Δ_4^{26}	0	0	0	0	0	0	0	1	0	0	2	2
Δ_4^{27}	0	0	0	0	0	0	0	1	0	2	0	0
Δ_4^{28}	0	0	0	0	0	0	0	0	0	1	0	0
Δ_4^{29}	0	0	0	0	0	0	2	1	0	0	2	0
Δ_4^{30}	0	0	0	0	0	0	0	1	2	0	2	0
Δ_4^{31}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{32}	0	0	0	0	0	0	2	0	0	0	0	2
Δ_4^{33}	0	0	0	0	0	0	0	0	0	1	0	0
Δ_4^{34}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{35}	0	0	0	0	0	0	0	0	0	0	0	2
Δ_4^{36}	0	0	0	0	0	0	1	0	0	0	0	0
Δ_4^{37}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_4^{38}	0	0	0	0	0	0	0	0	0	0	1	2
Δ_4^{39}	0	0	0	0	0	0	0	0	2	0	0	2
Δ_4^{40}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_4^{41}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_4^{42}	0	0	0	0	0	0	0	0	0	0	1	2
Δ_4^{43}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{44}	0	0	0	0	0	0	0	0	0	2	0	2
Δ_4^{45}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{46}	0	0	0	0	0	0	0	0	0	0	0	2
Δ_4^{47}	0	0	0	0	0	0	1	0	0	0	0	0
Δ_4^{48}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_4^{49}	0	0	0	0	0	0	0	0	0	1	0	0
Δ_4^{50}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_4^{51}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{52}	0	0	0	0	0	0	0	0	0	0	0	2
Δ_4^{53}	0	0	0	0	0	0	1	0	0	0	0	2
Δ_4^{54}	0	0	0	0	0	0	0	0	1	0	0	2
Δ_4^{55}	0	0	0	0	0	0	0	0	0	1	0	2
Δ_5^{1}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	0	0	0	0	0	0	0	0	0	0	0

	Δ_3^8	Δ_4^1	Δ_4^2	Δ_4^3	Δ_4^4	Δ_4^5	Δ_4^6	Δ_4^7	Δ_4^8	Δ_4^9	Δ_4^{10}	Δ_4^{11}
Δ_4^{24}	1	0	0	0	0	4	0	0	0	0	0	1
Δ_4^{25}	2	0	0	0	1	0	0	2	0	0	0	2
Δ_4^{26}	2	0	0	0	1	0	2	0	0	0	0	2
Δ_4^{27}	2	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{28}	0	0	0	0	0	4	0	0	1	0	0	0
Δ_4^{29}	8	0	0	0	1	4	2	2	0	0	0	0
Δ_4^{30}	2	0	0	0	0	0	0	2	0	0	2	0
Δ_4^{31}	1	0	0	0	0	0	0	0	0	0	0	6
Δ_4^{32}	0	0	0	0	2	0	0	0	0	0	0	0
Δ_4^{33}	0	0	0	0	0	0	0	0	3	0	0	0
Δ_4^{34}	1	0	0	0	0	0	0	0	0	4	0	0
Δ_4^{35}	1	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{36}	0	0	0	1	0	0	0	2	0	0	0	0
Δ_4^{37}	0	0	0	0	2	0	2	0	0	1	0	0
Δ_4^{38}	1	0	0	0	0	0	0	1	2	0	0	2
Δ_4^{39}	1	0	0	0	0	0	0	0	0	0	0	1
Δ_4^{40}	2	0	0	2	0	0	0	0	0	0	0	0
Δ_4^{41}	1	0	0	0	0	0	0	1	0	0	0	0
Δ_4^{42}	1	0	0	0	0	0	1	0	0	2	0	2
Δ_4^{43}	1	0	1	0	0	1	0	0	0	0	0	0
Δ_4^{44}	1	0	0	0	0	0	0	0	0	0	0	1
Δ_4^{45}	1	0	1	0	0	1	0	0	0	0	0	0
Δ_4^{46}	0	0	0	2	0	0	0	0	0	0	2	0
Δ_4^{47}	0	0	0	1	0	0	2	0	0	0	0	0
Δ_4^{48}	1	0	0	0	0	0	1	0	0	0	0	0
Δ_4^{49}	0	0	0	0	2	0	0	2	1	0	0	0
Δ_4^{50}	0	0	0	0	0	0	0	0	0	3	0	0
Δ_4^{51}	1	0	0	0	0	0	0	0	4	0	0	0
Δ_4^{52}	1	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{53}	0	0	0	0	0	0	1	1	0	0	1	0
Δ_4^{54}	0	0	0	0	0	1	0	0	0	1	0	0
Δ_4^{55}	0	0	0	0	0	1	0	0	1	0	0	0
Δ_5^1	0	0	0	2	3	0	0	0	0	0	0	0
Δ_5^2	0	1	0	1	1	0	0	0	0	0	0	0
Δ_5^3	0	0	0	0	1	0	2	2	0	0	2	0

	Δ_4^{12}	Δ_4^{13}	Δ_4^{14}	Δ_4^{15}	Δ_4^{16}	Δ_4^{17}	Δ_4^{18}	Δ_4^{19}	Δ_4^{20}	Δ_4^{21}	Δ_4^{22}	Δ_4^{23}
Δ_4^{24}	0	0	0	0	0	4	0	0	0	2	2	0
Δ_4^{25}	2	3	2	0	2	0	2	0	0	0	10	0
Δ_4^{26}	2	3	0	0	0	0	8	0	2	10	0	4
Δ_4^{27}	0	0	2	6	0	4	0	0	4	4	0	2
Δ_4^{28}	0	0	4	0	4	0	0	0	0	4	0	0
Δ_4^{29}	0	1	2	0	2	4	0	4	2	0	0	0
Δ_4^{30}	0	0	2	0	0	0	0	4	6	0	4	2
Δ_4^{31}	0	2	0	0	0	0	12	0	4	1	2	0
Δ_4^{32}	1	1	2	0	0	2	5	2	0	2	2	2
Δ_4^{33}	0	2	2	2	2	0	6	2	0	4	0	8
Δ_4^{34}	0	0	4	4	2	4	1	2	0	0	9	0
Δ_4^{35}	2	0	4	2	0	0	1	2	4	0	3	0
Δ_4^{36}	0	2	2	2	2	0	4	1	2	6	0	2
Δ_4^{37}	0	0	2	2	0	5	2	0	4	0	2	0
Δ_4^{38}	0	0	1	0	2	0	6	2	0	2	0	8
Δ_4^{39}	0	2	2	2	2	0	2	0	2	0	4	0
Δ_4^{40}	1	3	0	4	2	2	0	4	2	2	2	2
Δ_4^{41}	0	0	3	4	6	2	0	0	2	2	2	2
Δ_4^{42}	0	0	0	0	0	0	6	2	2	0	2	0
Δ_4^{43}	0	0	4	0	0	4	0	2	0	0	2	0
Δ_4^{44}	0	2	4	2	2	2	0	0	2	4	0	4
Δ_4^{45}	0	0	4	0	0	6	2	2	0	2	0	0
Δ_4^{46}	1	1	2	4	4	2	0	6	4	1	1	0
Δ_4^{47}	0	2	4	2	2	4	0	1	2	0	6	0
Δ_4^{48}	0	0	2	4	2	0	2	0	6	2	2	0
Δ_4^{49}	0	0	0	2	4	0	2	0	0	2	0	2
Δ_4^{50}	0	2	0	2	0	3	0	2	2	0	4	0
Δ_4^{51}	0	0	0	4	0	2	0	2	2	9	0	2
Δ_4^{52}	2	0	0	2	4	2	0	2	0	3	0	4
Δ_4^{53}	0	1	3	1	1	4	0	1	1	2	2	1
Δ_4^{54}	1	1	3	0	0	1	0	1	4	0	4	1
Δ_4^{55}	1	1	0	0	4	2	4	1	0	4	0	2
Δ_5^{1}	3	0	0	0	0	6	0	0	0	3	3	0
Δ_5^{2}	1	0	0	3	6	0	3	3	6	0	0	3
Δ_5^{3}	3	0	2	2	0	2	3	8	0	2	2	2

	Δ_4^{24}	Δ_4^{25}	Δ_4^{26}	Δ_4^{27}	Δ_4^{28}	Δ_4^{29}	Δ_4^{30}	Δ_4^{31}	Δ_4^{32}	Δ_4^{33}	Δ_4^{34}	Δ_4^{35}
Δ_4^{24}	0	2	2	0	4	4	0	0	0	4	12	0
Δ_4^{25}	2	9	10	0	0	4	4	8	2	0	0	4
Δ_4^{26}	2	10	9	2	0	4	0	2	2	8	0	4
Δ_4^{27}	0	0	2	3	4	2	0	0	4	0	0	0
Δ_4^{28}	4	0	0	4	4	4	0	0	4	6	4	4
Δ_4^{29}	4	4	4	2	4	11	0	0	6	0	0	4
Δ_4^{30}	0	4	0	0	0	0	17	4	4	0	4	8
Δ_4^{31}	0	8	2	0	0	0	4	4	10	0	0	0
Δ_4^{32}	0	1	1	2	2	3	2	5	6	6	2	2
Δ_4^{33}	2	0	4	0	3	0	0	0	6	7	0	0
Δ_4^{34}	6	0	0	0	2	0	2	0	2	0	9	6
Δ_4^{35}	0	2	2	0	2	2	4	0	2	0	6	9
Δ_4^{36}	2	2	0	4	4	0	0	0	0	4	2	4
Δ_4^{37}	2	0	2	0	0	0	2	2	4	0	2	4
Δ_4^{38}	0	3	3	0	0	1	0	6	4	4	0	2
Δ_4^{39}	1	2	0	4	2	0	4	0	8	0	4	6
Δ_4^{40}	0	2	2	0	2	0	2	0	0	4	0	4
Δ_4^{41}	4	1	3	2	0	1	0	2	4	0	4	4
Δ_4^{42}	0	3	3	1	0	1	8	6	4	6	4	2
Δ_4^{43}	2	0	4	4	6	0	0	2	4	2	4	2
Δ_4^{44}	1	0	2	2	0	0	0	2	8	6	2	0
Δ_4^{45}	2	4	0	4	4	0	0	0	4	6	2	6
Δ_4^{46}	4	1	1	2	2	7	0	0	1	0	8	6
Δ_4^{47}	2	0	2	2	0	0	2	4	0	4	6	2
Δ_4^{48}	4	3	1	3	2	1	2	0	4	2	6	2
Δ_4^{49}	2	2	0	2	5	0	0	2	4	10	6	2
Δ_4^{50}	2	4	0	2	0	0	8	6	6	6	8	6
Δ_4^{51}	6	0	0	4	4	0	0	1	2	8	2	2
Δ_4^{52}	0	2	2	4	0	2	0	1	2	6	2	12
Δ_4^{53}	0	1	1	3	4	5	1	0	1	3	4	3
Δ_4^{54}	3	0	1	0	2	2	2	4	6	3	8	6
Δ_4^{55}	3	1	0	3	1	2	1	0	6	5	1	3
Δ_5^{1}	0	3	3	0	6	3	0	0	6	0	6	0
Δ_5^{2}	6	0	0	0	0	0	3	3	6	6	3	3
Δ_5^{3}	4	3	3	2	2	5	2	3	2	2	0	8

	Δ_4^{36}	Δ_4^{37}	Δ_4^{38}	Δ_4^{39}	Δ_4^{40}	Δ_4^{41}	Δ_4^{42}	Δ_4^{43}	Δ_4^{44}	Δ_4^{45}	Δ_4^{46}	Δ_4^{47}
Δ_4^{24}	4	4	0	2	0	8	0	4	2	4	8	4
Δ_4^{25}	4	0	6	4	4	2	6	0	0	8	2	0
Δ_4^{26}	0	4	6	0	4	6	6	8	4	0	2	4
Δ_4^{27}	8	0	0	8	0	4	2	8	4	8	4	4
Δ_4^{28}	8	0	0	4	4	0	0	12	0	8	4	0
Δ_4^{29}	0	0	2	0	0	2	2	0	0	0	14	0
Δ_4^{30}	0	4	0	8	4	0	16	0	0	0	0	4
Δ_4^{31}	0	4	12	0	0	4	12	4	4	0	0	8
Δ_4^{32}	0	4	4	8	0	4	4	4	8	4	1	0
Δ_4^{33}	4	0	4	0	4	0	6	2	6	6	0	4
Δ_4^{34}	2	2	0	4	0	4	4	4	2	2	8	6
Δ_4^{35}	4	4	2	6	4	4	2	2	0	6	6	2
Δ_4^{36}	5	0	4	6	0	4	4	8	4	0	0	2
Δ_4^{37}	0	5	2	6	4	6	6	4	4	4	4	12
Δ_4^{38}	4	2	10	0	4	3	10	2	4	2	2	4
Δ_4^{39}	6	6	0	7	0	6	4	2	8	6	4	4
Δ_4^{40}	0	4	4	0	13	4	4	8	0	8	10	0
Δ_4^{41}	4	6	3	6	4	4	4	6	4	4	2	4
Δ_4^{42}	4	6	10	4	4	4	10	2	0	2	2	4
Δ_4^{43}	8	4	2	2	8	6	2	5	6	8	0	0
Δ_4^{44}	4	4	4	8	0	4	0	6	7	2	4	6
Δ_4^{45}	0	4	2	6	8	4	2	8	2	5	0	8
Δ_4^{46}	0	4	2	4	10	2	2	0	4	0	12	0
Δ_4^{47}	2	12	4	4	0	4	4	0	6	8	0	5
Δ_4^{48}	4	6	4	4	4	2	3	4	6	6	2	4
Δ_4^{49}	12	4	6	4	4	6	2	4	6	4	4	0
Δ_4^{50}	4	10	6	6	4	2	4	6	0	2	0	4
Δ_4^{51}	6	6	4	2	0	6	0	2	4	4	8	2
Δ_4^{52}	2	2	2	0	4	2	2	6	6	2	6	4
Δ_4^{53}	4	2	1	4	8	3	1	6	4	6	10	4
Δ_4^{54}	4	2	3	5	3	4	4	4	4	3	1	6
Δ_4^{55}	6	6	4	4	3	3	3	3	5	4	1	4
Δ_5^{1}	6	6	0	0	0	6	0	6	0	6	0	6
Δ_5^{2}	6	0	12	3	3	3	12	3	3	3	0	6
Δ_5^{3}	2	2	4	0	4	6	4	2	0	2	2	2

	Δ_4^{48}	Δ_4^{49}	Δ_4^{50}	Δ_4^{51}	Δ_4^{52}	Δ_4^{53}	Δ_4^{54}	Δ_4^{55}	Δ_5^{1}	Δ_5^{2}	Δ_5^{3}
Δ_4^{24}	8	4	4	12	0	0	12	12	0	8	8
Δ_4^{25}	6	4	8	0	4	4	0	4	2	0	6
Δ_4^{26}	2	0	0	0	4	4	4	0	2	0	6
Δ_4^{27}	6	4	4	8	8	12	0	12	0	0	4
Δ_4^{28}	4	10	0	8	0	16	8	4	4	0	4
Δ_4^{29}	2	0	0	0	4	20	8	8	2	0	10
Δ_4^{30}	4	0	16	0	0	4	8	4	0	4	4
Δ_4^{31}	0	4	12	2	2	0	16	0	0	4	6
Δ_4^{32}	4	4	6	2	2	2	12	12	2	4	2
Δ_4^{33}	2	10	6	8	6	6	6	10	0	4	2
Δ_4^{34}	6	6	8	2	2	8	16	2	2	2	0
Δ_4^{35}	2	2	6	2	12	6	12	6	0	2	8
Δ_4^{36}	4	12	4	6	2	8	8	12	2	4	2
Δ_4^{37}	6	4	10	6	2	4	4	12	2	0	2
Δ_4^{38}	4	6	6	4	2	2	6	8	0	8	4
Δ_4^{39}	4	4	6	2	0	8	10	8	0	2	0
Δ_4^{40}	4	4	4	0	4	16	6	6	0	2	4
Δ_4^{41}	2	6	2	6	2	6	8	6	2	2	6
Δ_4^{42}	3	2	4	0	2	2	8	6	0	8	4
Δ_4^{43}	4	4	6	2	6	12	8	6	2	2	2
Δ_4^{44}	6	6	0	4	6	8	8	10	0	2	0
Δ_4^{45}	6	4	2	4	2	12	6	8	2	2	2
Δ_4^{46}	2	4	0	8	6	20	2	2	0	0	2
Δ_4^{47}	4	0	4	2	4	8	12	8	2	4	2
Δ_4^{48}	4	6	0	4	4	6	6	8	2	2	6
Δ_4^{49}	6	5	0	2	4	4	12	4	2	0	2
Δ_4^{50}	0	0	7	0	0	6	10	6	0	4	2
Δ_4^{51}	4	2	0	9	6	8	2	16	2	2	0
Δ_4^{52}	4	4	0	6	9	6	6	12	0	2	8
Δ_4^{53}	3	2	3	4	3	10	7	7	2	0	7
Δ_4^{54}	3	6	5	1	3	7	10	9	2	2	1
Δ_4^{55}	4	2	3	8	6	7	9	10	2	2	1
Δ_5^{1}	6	6	0	6	0	12	12	12	4	6	3
Δ_5^{2}	3	0	6	3	3	0	6	6	3	5	3
Δ_5^{3}	6	2	2	0	8	14	2	2	1	2	12

4 Sub-Orbit Representatives

4.1 $G \cong Th, t \in X = 2A$

Conjugating elements are given as words in the standard generators of G, so that $a \in 2A, b \in 3A$ and $ab \in 19A$. We take $t = a$, and then the fourth column below contains $g \in G$ such that t^g belongs to the stated $C_G(t)$-orbit.

$C_G(t)$-orbit	z class	Size	Conjugating Word
$\Delta_0^1(t)$	$1A$	1	-
$\Delta_1^1(t)$	$2A$	270	t^{g_1}
$\Delta_2^1(t)$	$2A$	30240	bab^2abab^2ab
$\Delta_2^2(t)$	$4A$	34560	$g_1 = b^2abab^2ab^2abababab^2ab$
$\Delta_3^1(t)$	$3A$	61440	bab^2ab^2ab
$\Delta_3^2(t)$	$4A$	483840	$b^2ab^2abab^2abab^2abab$
$\Delta_3^3(t)$	$4B$	2903040	$bab^2ababab^2abab^2(ab)^3$
$\Delta_3^4(t)$	$6B$	3870720	$b^2(ab)^9$
$\Delta_3^5(t)$	$8A$	3870720	$babab^2ab$
$\Delta_3^6(t)$	$8A$	3870720	bab^2abab
$\Delta_4^1(t)$	$3C$	430080	$(b^2a)^3babab$
$\Delta_4^2(t)$	$3B$	573440	$babab^2ab^2abab$
$\Delta_4^3(t)$	$6A$	3870720	$babab$
$\Delta_4^4(t)$	$5A$	4644864	b^2ab
$\Delta_4^5(t)$	$9A$	5160960	$bab^2(ab)^3$
$\Delta_4^6(t)$	$6C$	15482880	$bab^2ababab^2ab$
$\Delta_4^7(t)$	$7A$	15482880	$(ba)^5b$
$\Delta_4^8(t)$	$9C$	15482880	bab
$\Delta_4^9(t)$	$9C$	15482880	$b^2abab^2(ab)^3$
$\Delta_4^{10}(t)$	$12C$	15482880	$g_2 = (b^2a)^2(ba)^4b$
$\Delta_4^{11}(t)$	$12C$	15482880	g_2^{-1}
$\Delta_4^{12}(t)$	$10A$	23224320	b
$\Delta_4^{13}(t)$	$13A$	30965760	$bab^2a(ba)^3b$
$\Delta_4^{14}(t)$	$12D$	46448640	$(b^2a)^2(ba)^5b$
$\Delta_4^{15}(t)$	$14A$	46448640	b^2abab^2abab
$\Delta_4^{16}(t)$	$18A$	46448640	$bab^2ab^2abab^2ab$
$\Delta_4^{17}(t)$	$18B$	46448640	$g_3 = b^2(ab)^6$
$\Delta_4^{18}(t)$	$18B$	46448640	g_3^{-1}
$\Delta_4^{19}(t)$	$20A$	46448640	$g_4 = (b^2a)^2(ba)^3b^2ab$
$\Delta_4^{20}(t)$	$20A$	46448640	g_4^{-1}
$\Delta_4^{21}(t)$	$28A$	46448640	$g_5 = bab^2(ab)^5$
$\Delta_4^{22}(t)$	$28A$	46448640	g_5^{-1}
$\Delta_4^{23}(t)$	$36A$	46448640	$g_6 = b^2(ab)^3ab^2ab$
$\Delta_4^{24}(t)$	$36A$	46448640	g_6^{-1}
$\Delta_4^{25}(t)$	$19A$	92897280	b^2abab
$\Delta_4^{26}(t)$	$21A$	92897280	$b^2(ab)^3$
$\Delta_5^1(t)$	$9B$	10321920	$ba(b^2a)^4babab$
$\Delta_5^2(t)$	$27A$	92897280	$b^2(ab)^4$

4.2 $G \cong HN, t \in X = 2B$

Orbits are supplied with either a word which conjugates t to a representative for that orbit, or a symbol of the form $g \to n$ ($n \in \mathbb{N}, g \in G$, where g represents a word from elsewhere in the table) which denotes that a representative may be obtained by 'powering down' from the relevant orbit, so that a representative is given by $t(tt^g)^n$.

Again words are in the standard generators of G, so $a \in 2A, b \in 3B$, ab has order 22 and $abab^2$ has order 5. We take $t = (abab^2ab)^{10} \in 2B$.

$C_G(t)$-orbit	z class	Size	Conjugating Word
$\Delta_0^1(t)$	$1A$	1	-
$\Delta_1^1(t)$	$2B$	150	$g_1 \to 4$
$\Delta_2^1(t)$	$2A$	960	$g_2 \to 10$
$\Delta_2^2(t)$	$2B$	7200	$g_3 \to 5$
$\Delta_2^3(t)$	$4A$	9600	$g_1 \to 2$
$\Delta_3^1(t)$	$3A$	25600	$g_4 \to 4$
$\Delta_3^2(t)$	$4A$	115200	$g_5 \to 5$
$\Delta_3^3(t)$	$4B$	115200	$g_2 \to 5$
$\Delta_3^4(t)$	$4C$	115200	$g_6 \to 5$
$\Delta_3^5(t)$	$4C$	115200	$g_7 \to 5$
$\Delta_3^6(t)$	$6A$	230400	$g_8 \to 2$
$\Delta_3^7(t)$	$8B$	460800	$g_1 = bab^2a(ba)^2b^2a(ba)^3$
$\Delta_3^8(t)$	$8B$	460800	$(ba)^3b^2a(ba)^2b^2aba$
$\Delta_4^1(t)$	$5A$	15360	$g_2 \to 4$
$\Delta_4^2(t)$	$5B$	36864	$g_9 \to 5$
$\Delta_4^3(t)$	$3B$	102400	$g_{10} \to 10$
$\Delta_4^4(t)$	$6A$	153600	$g_{11} \to 2$
$\Delta_4^5(t)$	$10A$	184320	$(ba)^4(b^2a)^3(ba)^2b^2ab^2$
$\Delta_4^6(t)$	$5E$	184320	$g_{12} \to 2$
$\Delta_4^7(t)$	$5E$	184320	$g_{13} \to 2$
$\Delta_4^8(t)$	$5CD$	184320	$g_{10} \to 6$
$\Delta_4^9(t)$	$5CD$	184320	$g_{14} \to 6$
$\Delta_4^{10}(t)$	$5E$	184320	$g_3 \to 2$
$\Delta_4^{11}(t)$	$10A$	184320	$((ab)^2ab^2)^2(ab)^4ab^2(ab)^2$
$\Delta_4^{12}(t)$	$10B$	307200	$g_2 \to 2$
$\Delta_4^{13}(t)$	$6B$	460800	$g_4 \to 2$
$\Delta_4^{14}(t)$	$10F$	921600	$g_{12} = (ba)^2(b^2a)^2b^2$
$\Delta_4^{15}(t)$	$12A$	921600	$g_8 = (ba)^3b^2(ab)^2$
$\Delta_4^{16}(t)$	$12B$	921600	$g_4 = ab^2(ab)^4$
$\Delta_4^{17}(t)$	$10DE$	921600	$g_{10} \to 3$
$\Delta_4^{18}(t)$	$20AB$	921600	$g_5 = babab^2$
$\Delta_4^{19}(t)$	$6C$	921600	$g_{10} \to 5$

$\Delta_4^{20}(t)$	$12B$	921600	$(ab)^4ab^2$
$\Delta_4^{21}(t)$	$20AB$	921600	$bab^2a(ba)^4b$
$\Delta_4^{22}(t)$	$20AB$	921600	$(ab)^2ab^2(ab)^4$
$\Delta_4^{23}(t)$	$10GH$	921600	$g_3 = (ba)^4(b^2a)^2(ba)^3$
$\Delta_4^{24}(t)$	$10C$	921600	$g_5 \to 2$
$\Delta_4^{25}(t)$	$10GH$	921600	$(ab)^5(ab^2)^2$
$\Delta_4^{26}(t)$	$10GH$	921600	$bab^2a(ba)^6b^2$
$\Delta_4^{27}(t)$	$10F$	921600	$g_{13} = (ba)^3b^2ab^2$
$\Delta_4^{28}(t)$	$10DE$	921600	$g_{14} \to 3$
$\Delta_4^{29}(t)$	$12A$	921600	$g_{11} = (ba)^6b$
$\Delta_4^{30}(t)$	$10GH$	921600	$(ab^2)^2(ab)^5$
$\Delta_4^{31}(t)$	$20AB$	921600	bab^2ab^2
$\Delta_4^{32}(t)$	$30A$	1843200	$g_{17} = ba(b^2a)^3(ba)^3b$
$\Delta_4^{33}(t)$	$12C$	1843200	$(ba)^8b$
$\Delta_4^{34}(t)$	$30BC$	1843200	$g_{10} \to 7$
$\Delta_4^{35}(t)$	$15BC$	1843200	$g_{10} \to 14$
$\Delta_4^{36}(t)$	$20DE$	1843200	$g_7 \to 3$
$\Delta_4^{37}(t)$	$20DE$	1843200	$g_7 = (ba)^2b^2aba$
$\Delta_4^{38}(t)$	$22A$	1843200	$g_{16} = (ab)^4ab^2(ab)^2a$
$\Delta_4^{39}(t)$	$15BC$	1843200	$g_{14} \to 2$
$\Delta_4^{40}(t)$	$20C$	1843200	$g_2 = (ba)^3b^2a(ba)^5b^2$
$\Delta_4^{41}(t)$	$11A$	1843200	$g_{16} \to 2$
$\Delta_4^{42}(t)$	$22A$	1843200	$g_{18} = (ba)^2b^2a(ba)^3b$
$\Delta_4^{43}(t)$	$30BC$	1843200	$g_{14} = (ba)^4b$
$\Delta_4^{44}(t)$	$15BC$	1843200	$g_{10} \to 2$
$\Delta_4^{45}(t)$	$30BC$	1843200	$g_{10} = (ab)^2(ab^2)^2(ab)^5$
$\Delta_4^{46}(t)$	$15A$	1843200	$g_{17} \to 2$
$\Delta_4^{47}(t)$	$20DE$	1843200	$g_6 = bab^2(ab)^2$
$\Delta_4^{48}(t)$	$11A$	1843200	$g_{18} \to 2$
$\Delta_4^{49}(t)$	$20DE$	1843200	$g_6 \to 3$
$\Delta_4^{50}(t)$	$12C$	1843200	$(ab)^2ab^2(ab)^5$
$\Delta_4^{51}(t)$	$30BC$	1843200	$g_{19} = (ba)^7b^2(ab)^2$
$\Delta_4^{52}(t)$	$15BC$	1843200	$g_{19} \to 2$
$\Delta_4^{53}(t)$	$21A$	3686400	$g_{20} = bab^2a(ba)^4$
$\Delta_4^{54}(t)$	$25AB$	3686400	$g_9 = bab$
$\Delta_4^{55}(t)$	$25AB$	3686400	$g_9 \to 2$
$\Delta_5^{1}(t)$	$7A$	614400	$g_{20} \to 3$
$\Delta_5^{2}(t)$	$9A$	1228800	$bab^2a(ba)^2b^2a(ba)^4b$
$\Delta_5^{3}(t)$	$14A$	1843200	$(ba)^2(b^2a)^2bab$

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