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Shape corrections for 3D EIT

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Abstract. Movement of the boundary in biomedical Electrical Impedance Tomography (EIT) has been always a source of error in image reconstruction. In the case of pulmonary EIT, where the patient's chest shape changes during respiration, this is inevitable, so it is essential to be able to correct for shape changes and consequently avoid artifacts. Assuming that the conductivity is isotropic, an assumption that is reasonable for lung tissue but admittedly violated for muscle, the boundary shape up to a Möbius transformation (conformal mapping) as well as the conductivity can theoretically be determined by 3D EIT data. While in two dimensions the space of conformal mappings are infinite dimensional, in the three dimensional case the Möbius transformations are given by a finite number of parameters. In this paper, we concentrate on the three dimensional case and take a linear approximation. We will give results of numerical studies analogous to the two dimensional work of Boyle et al on the effect of electrode movement and shape error in 3D EIT.

1. Introduction

Movement of the boundary in EIT causes artifacts in the images. This issue arises more often in pulmonary EIT, where the patient's chest moves continuously due to breathing and posture changes [1]. During the last years some improvement has been made towards a better reconstruction when deformation of the boundary occurs. Lionheart [2] has shown that non-conformal changes in boundary shapes and electrode locations can be uniquely determined. Moreover, Soleimani et al. [3] showed that in some cases a combination of image reconstruction model of both conductivity and shape changes can be used to recover conductivity and shape changes.

Generally, distortions of a domain spoil the assumption of an isotropic conductivity distribution [2], but this is not always the case. The distortions that are conformal maps, that is functions which preserve angles, do not lead to an anisotropic conductivity [4]. In two dimensions there are infinite conformal maps, while in three dimensions these maps are a finite dimensional set, known widely as the Möbius transformations, and their linearizations, the infinitesimal Möbius transformations.

In this paper, we present numerical results supporting the idea of recovering the non-conformal part of the linearized distortion from the EIT data. Our results show that reconstruction with shape correction gives a distorted conductivity of the true isotropic conductivity.

2. Conformal Vector Fields

Let $\mathbf{V}$ be a sufficiently smooth vector field. A distortion can be linearized by adding a vector field $\mathbf{V}$ to each point. In the case of distortions that preserve the angles (that is, conformal mappings) then $\mathbf{V}$ is called conformal vector field. A vector field $\mathbf{V}$ is conformal if and only if

$$\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} = (\nabla \cdot \mathbf{V})\, \delta_{i,j}. \tag{1}$$

is satisfied. In two dimensions it is straight forward to show that (1) satisfies the Cauchy-Riemann equations. As we have mentioned, in the three dimensional space there is a finite dimensional set of conformal mappings, the Möbius transformations.

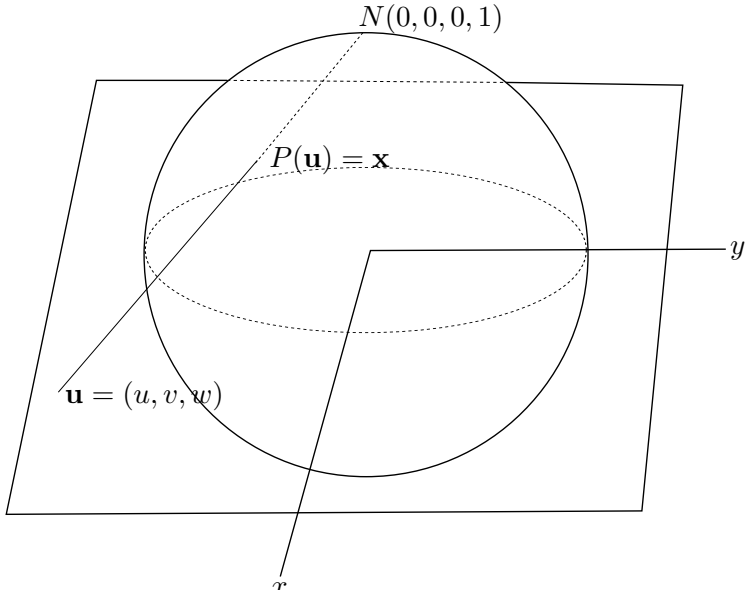


Figure 1: Stereographic projection

Consider the 4-dimensional sphere S^3. Using stereographic projection from the hypersphere on $\mathbb{R}^3$,then applying a rotation and finally backprojecting we construct a conformal vector field, since both stereographic projections and rotations preserve angles.

Let $S^3 \subset \mathbb{R}^4$ be the 4-dimensional sphere and $N(0,0,0,1)$ be the north pole. Suppose $(x,y,z,t) \in S^3$ is an arbitrary point on the hypersphere. It is easy to calculate the stereographic projection $P : \mathbb{R}^3 \to S^3 \setminus \{N\}$, $(u,v,w) \mapsto (x,y,z,t)$

$$\begin{cases} x = \frac{2u}{u^2+v^2+w^2+1} \\ y = \frac{2v}{u^2+v^2+w^2+1} \\ z = \frac{2w}{u^2+v^2+w^2+1} \\ t = \frac{u^2+v^2+w^2-1}{u^2+v^2+w^2+1} \end{cases} \tag{2}$$

Let $T : S^3 \setminus \{N\} \to \mathbb{R}^3$ denote the inverse mapping. Let $R_\theta \in SO(4)$ be the rotation matrix about the $x-t$ hyperplane and let F_θ represent a curve, where θ is the angle of rotation. Then,

$$F_\theta = T R_\theta S(\mathbf{x}), \tag{3}$$

where $\mathbf{x} = (x,y,z,t) \in S^3$. A Möbius transformation vector is given by

$$\left.\frac{dF_\theta}{d\theta}\right|_{\theta=0} = DT \left.\frac{dR_\theta}{d\theta}\right|_{\theta=0} S(\mathbf{x}) = \begin{pmatrix} \frac{-u^2+v^2+w^2-1}{2} \\ -uv \\ -uw \end{pmatrix}. \tag{4}$$

where DT is the Jacobian of the inverse mapping.

Permuting the axes generates other independent Möbius vector fields, and we can thus construct a basis of the conformal vector fields. The theory predicts that any vector field orthogonal to the space of Möbius vector fields can be determined from complete boundary data while a distortion by a Möbius vector field will produce data for which there is a consistent isotropic conductivity, which is distorted from the true one by the Möbius vector field.

As a first test of this in the context of finitely many electrodes and a finite element mesh we will simply verify that data subject to a conformal distortion results in a recognisable but distorted reconstruction of the true conductivity but a non-conformal distortion of the same size produces a reconstruction with more artifacts.

3. Numerical Experiments

In order to verify the theoretical results we used the familiar demonstration model demo_real from EIDORS [5]. Specifically, we used a finite element mesh with electrode positions (green) and simulated inhomogeneities (blue and red) (Figure 2). For our tests we modified the existing code to be able to apply conformal and non-conformal distortions on the inhomogeneities.

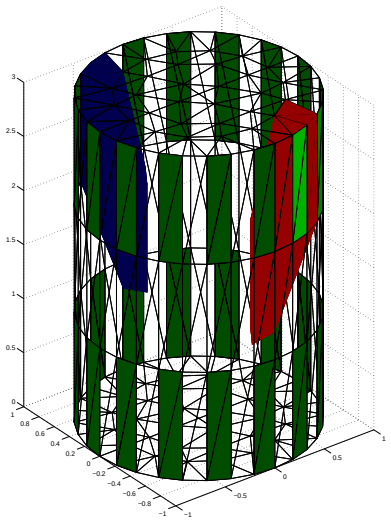


Figure 2: Finite element mesh with electrodes and simulated inhomogeneities.

In detail, we started by applying the following non-conformal perturbation

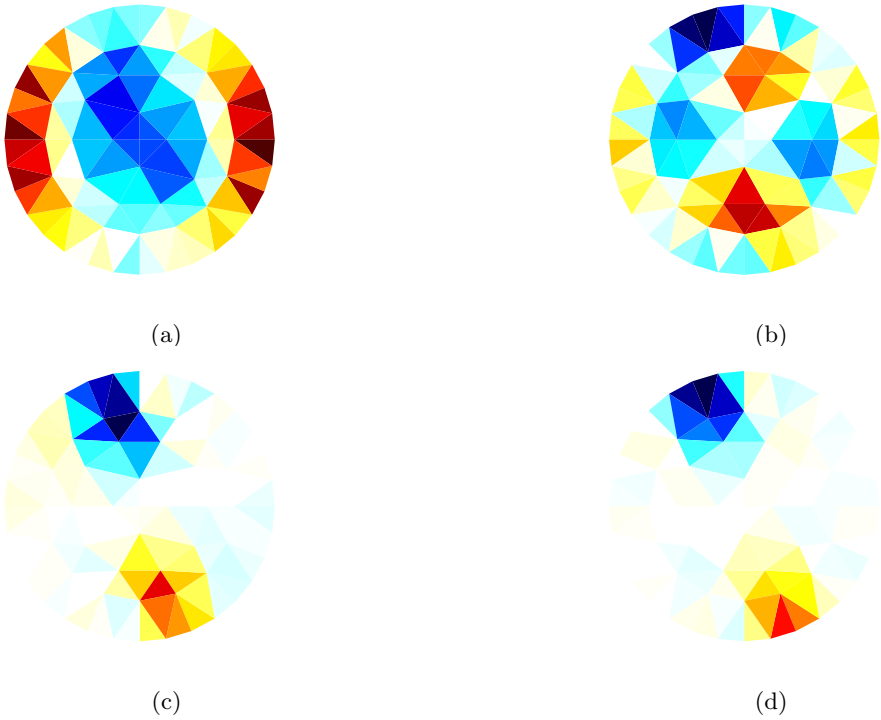
$$(x, y, z) \to \left(x + \epsilon \left(x \frac{z+3}{3} \right), y + \epsilon \left(y \frac{-z+6}{3} \right), z \right), \tag{5}$$

where ϵ is small enough and it was chosen so that the L^2-norms of the perturbations are equal. The non-conformal transformation was chosen so that it transforms circles to ellipses, that is a transformation which is "very" non-conformal. For the conformal distortion we used the Möbius vector (4) we have constructed before. The applied perturbation is given by

$$(x, y, z) \to \left(x + \epsilon \frac{-x^2 + y^2 + z^2 - 1}{2}, y - \epsilon xy, z - \epsilon xz \right). \tag{6}$$

The reconstruction was a standard regularized linear reconstruction using the undistorted mesh.

The simulated results shown in Figure 3 confirm the theory. It is clear that non-conformal movements cause significant artifacts in the conductivity reconstruction. On the other hand, conformal distortions do not affect the conductivity reconstruction, at least not as importantly as with non-conformal distortions.



(a) (b)

(c) (d)

Figure 3: Reconstructed conductivity distributions at z=1 (*left*) and z=2 *right*. *Top* (3a), (3b): Non-conformal distortion applied. *Bottom* (3c), (3d) Conformal distortion applied.

4. Discussion and Conclusion

This paper deals with the effect of conformal and non-conformal distortions on the conductivity reconstruction in Electrical Impedance Tomography. The simulated results show an important difference in the conductivity reconstruction for the two distortions and suggest that conformal vector fields (Möbius transformations) tend to give a better reconstruction, avoiding artifacts.

It is hoped that the idea in this paper can be used to reduce artifacts in chest EIT images caused by variable chest shape. The idea is to start with an initial realistic chest shape, but to compensate for the breathing component by calculating a non-conformal shape perturbation which is adjusted along with the conductivity to fit the data at each time frame. The error will be a conformal map, which can either be determined by a small number of mechanical measurements, or if undetermined will result in a distortion of the conductivity image that will still be clinically useful.

Acknowledgments

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