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Ranking Boards of Directors

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Abstract

We propose ranking the importance (centrality) of boards of directors using a variant of the PageRank algorithm from computational graph theory. PageRank is at the heart of the immensely successful Google web search engine and, we argue, can be naturally extended to social network settings. Using tools from graph theory, we represent the board of directors as an undirected bipartite graph. Since PageRank operates on directed graphs we develop a procedure to pass from an undirected bipartite graph to appropriately weighted, directed projections. Finally, to illustrate the procedure we present preliminary rankings of publicly traded US and UK firms using this method. Our procedure provides a quick and efficient way to rank the importance of corporate boards.

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1 Introduction

In this paper we propose a method for ranking the importance of boards of directors using techniques from computational graph theory.¹ The structural properties of networks, the domain of graph theory, is becoming an important research theme within the field of corporate governance. For example, Bebchuk et al. (2007) investigate the centrality of CEOs, Cohen et al. (2007) discuss the implications of networks on mutual fund returns, and recent research has analyzed social networks in the boardroom (e.g., Stathopoulos and Kirchmaier (2006), Kramarz and Thesmar (2006) and Conyon and Muldoon (2006*b*)). A particularly salient stream of network analysis has focused on the properties of vertices within networks.² A large number of so-called *centrality* measures have arisen which gauge the structural importance of a vertex relative to other vertices within a complex web of associations. Such centrality measures have been very useful in the field of social networks for understanding the roles played by different actors (Newman (2005), Wasserman and Faust (1994)). In this paper we describe an alternative measure of centrality. It is the PageRank algorithm from computational graph theory originally described in Brin and Page (1998) and Brin et al. (1999).

PageRank was designed for, and is at the heart of, the immensely popular Google search engine. The PageRank algorithm, as originally conceived, is a system designed to rank the overall importance of each page on the web—thus providing an index to order responses to a user’s web search. The rank that Google uses can, in addition to its interpretation in terms of random walks, be interpreted as a measure of a web page’s authority, overall importance, or influence: Higham and Taylor (2003) give a very clear exposition of both these interpretations, as well as an excellent introduction to the PageRank algorithm itself. Although the algorithm was originally designed to determine the importance of web pages, it can be naturally extended to other network settings, including social networks. In this paper our interest is in corporate governance networks, particularly the board of directors.

This paper makes a number of contributions to the extant corporate governance literature. First, we propose measuring the importance of the board of directors (i.e. centrality) using a variety of related measures computed with the PageRank algorithm. These measures assign a numeric value to each vertex in a graph. The rank assigned to a board derives from a) the numeric measure

¹An earlier version of this paper appeared in Conyon and Muldoon (2006*a*).

²One can think of a vertex as the basic unit of analysis in a network. For example, it can represent an individual, a board, an accounting firm, an organization etc. In the sociology, and social network literature, a vertex is often referred to as an *actor*.

intrinsic to the board and which one may choose to be the same for all boards, or, alternatively, which may be calibrated to reflect differences in firm attributes (such as market value, performance etc.) and b) a poll of the board’s ‘neighbors’ (those firms with which it is interlocked), weighted according to both the rank (i.e. esteem or influence) in which the neighboring board is held, as well as the relative ‘strength’ of the connection between the boards. In essence, contributions from boards that are themselves held in high esteem count more towards one’s own rank. A board’s rank is defined recursively and, as we will show in Section 3, it can be interpreted as a steady state consensus about which boards are deemed important.

Second, we show how the board of directors may be represented as a weighted, directed graph. PageRank is designed to operate on directed graphs (these and other graph-theoretic terms are defined carefully in Section 2), while the usual representation of the social network of corporate governance—as an affiliation network or bipartite graph—captures only symmetric, or non-directional relationships and so gives rise to undirected graphs. Accordingly, one of the challenges in the development of a measure one might term “BoardRank” is to find a way to incorporate extra information into the construction of the social network in such a way as to yield directed graphs. In Section 4 we propose such a method.

Third, we contribute to the empirical corporate governance literature by presenting preliminary empirical evidence on the centrality of boards. In Section 5, we apply the ranking method to data on the corporate governance of firms in the United States and in the United Kingdom to illustrate the procedure. Our method is not, however, limited to such data and should be useful to social network researchers who wish to develop PageRank-like centrality measures for arbitrary affiliation networks. The data used in Section 5 are, for the US, a snap shot of the board memberships of approximately 1,700 publicly traded firms in early 2003. The data for the UK, collected in 2002, are a snap shot of approximately 2,200 publicly traded firms. We provide some preliminary evidence on which firms receive the highest rank using the PageRank procedure for this pilot study. We also present a simple statistical model illustrating which firm-level factors (such as size, company performance etc.) help determine the rank of a firm.

Overall, our paper contributes to a growing accounting, finance and economics literature focusing on the phenomenon of social networks, especially in the board of directors (e.g., Kramarz and Thesmar (2006), Guedj and Barnea (2007), Stathopoulos and Kirchmaier (2006), Hallock (1997), Cohen et al. (2007), Bebchuk et al. (2007), Conyon and Muldoon (2006*b*)) and more generally

on boards and corporate governance (e.g., Lin et al. (2003), Young (2000), Peasnell et al. (2003), Lin et al. (2007)). The rest of this paper is organized as follows. In Section 2 we introduce some notions from the mathematical theory of graphs as applied to social networks and boards of directors. Then, in Section 3, we discuss the PageRank algorithm and touch on its application to social networks. In section 4 we describe a method to pass from unweighted bipartite graphs to weighted, directed projections. In section 5 we apply PageRank to board data from the United States & the United Kingdom and, finally, in section 6 we offer some concluding remarks.

2 Corporate boards and social networks

In this section we define terms and briefly review some important features of graph theory as applied to social networks. A much more extensive discussion of graphs and their representation, manipulation and application to the social sciences appears in Wasserman and Faust (1994). The theory of random graphs is given in Bollobás (2001). Newman (2003) gives an excellent review of recent developments in the field of complex systems. Newman et al. (2002) discuss specifically the application of random graphs to social networks. Newman (2005) discusses centrality measures based on random walks.

2.1 Basic terminology

A *network* (or *graph*) is a set of items termed *vertices* (or *nodes*) with connections between them called *edges*. In discussions about graphs representing social networks the nodes are sometimes also called *actors*. We will restrict our attention to networks derived from the world of corporate directorship and adopt the following conventions: our nodes will be of two types, either boards or the directors who sit on them. Edges will represent, among other things, membership of a board (in one sort of graph) or an interlock between boards (in another, related sort of graph).

The latter relationship, an interlock due to a shared director, is clearly a symmetric one: if board A is interlocked with board B, then B is automatically interlocked with A as well. This sort of reciprocal connection will be represented by an *undirected* edge. But we will also need the notion of a *directed* edge, which will represent a unidirectional connection. A typical example of such a relationship is “has influence on”: one can easily imagine a setting in which firm A has influence on firm B—perhaps because A is a major shareholder in B—but this connection is not symmetric:

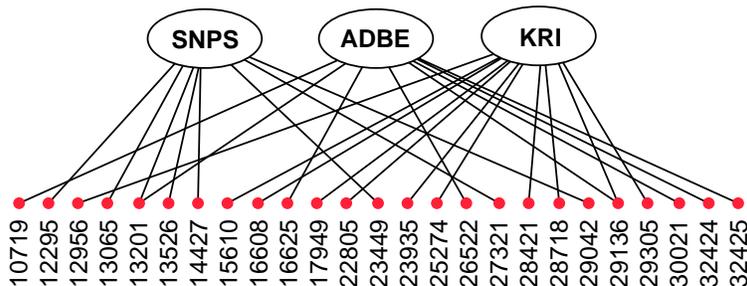


Figure 1: A *undirected, unweighted bipartite graph* representing the boards of directors of Adobe Systems (*ADBE*) and Synopsis (*SNPS*), both software houses, and that of the Knight Ridder (*KRI*) chain of newspapers.

there is no reason to imagine that B has influence on A. Both directed and undirected edges can also carry *weights*. In social networks these weights are usually a measure of the relative strength of the connection the edge represents and so a weight of zero is often taken to mean that the edge does not exist—that the corresponding connection is absent. We will use weights of this kind in PageRank algorithm (see Section 3), but we will also want consider another sort of weight—something one might term a “Boolean weight”—whose value is somewhat akin to that of a dummy variable in statistical modeling. These weights, whose role is discussed in Section 3.1, assume the values zero and one, but a Boolean weight of zero does not indicate the absence of the corresponding edge.

Finally, a graph whose every edge is directed is called a *directed graph*. If, in addition, all the edges have weights the graph is said to be a *weighted, directed graph*. Alternatively, a graph (such as the one pictured in Figure 1) in which all the edges are undirected and none of them have weights is an *unweighted, undirected graph* or, for short, an *undirected graph*.

2.1.1 Notation for edges and weights

Throughout this paper we will write $e_{j,k}$ to refer to a directed edge that connects vertex k (at the tail) to vertex j (at the tip). When there is a weight associated with the edge we will call it $w_{j,k}$. We will use similar notations, $E_{j,k}$ and $W_{j,k}$, for *undirected* edges. Of course, in this latter case $E_{1,2}$ is the same as $E_{2,1}$ and so $W_{1,2} = W_{2,1}$.

2.1.2 Paths, length and connectedness

A directed graph is said to contain a *path* between vertices a and b if it contains a sequence of edges $e_{a,k_1}, e_{k_1,k_2}, \dots, e_{k_l,b}$. That is, there is a path from a to b if one can get from one to the other by moving along the directed edges of the graph. A similar definition applies to undirected graphs. In either case, the *length* of the path is the total number of edges involved.

In an undirected graph two vertices are *connected* if there is a path between them and the *connected component* associated with a vertex is that part of the graph consisting of the vertex itself and all those others that can be reached by paths running along the edges of the graph. In a directed graph the notions of connectivity are slightly more complicated: node a is said to be *reachable* from node b if there is a path from b to a . But a has two, possibly distinct, connected components: those nodes reachable from a and those from which a can be reached.

2.1.3 Degree

The concept of *degree* will prove important. At its simplest, in an undirected graph, a node's degree is the just number of edges connected to it. By contrast, a node j appearing in a directed graph has both an *in degree* (the number of directed edges having j at their tips) and an *out degree* (the number of edges with j at their tails). These notions are generalized further, to accommodate weighted edges, in section 3 below.

2.2 Graphs of boards and directors

Data about boards of directors present an immediate problem: how should one draw a graph to represent it? The issue is that one could treat the board as the basic unit of analysis and form a graph whose vertices represent boards and whose edges represent interlocks (that is, shared directors). But alternatively, one could focus on the director and make a (generally much larger) graph whose vertices represent directors and whose edges represent shared board memberships. There is no obvious way to choose between these two representations and many authors simply analyze both. In Section 5 we will focus on the board-and-interlock graph, though our methods are equally applicable to the graph in which directors are vertices.

But the ambiguity about representation of the corporate world arises from the structure of the data: there really are two sorts of social entities here, the directors and the boards, and the

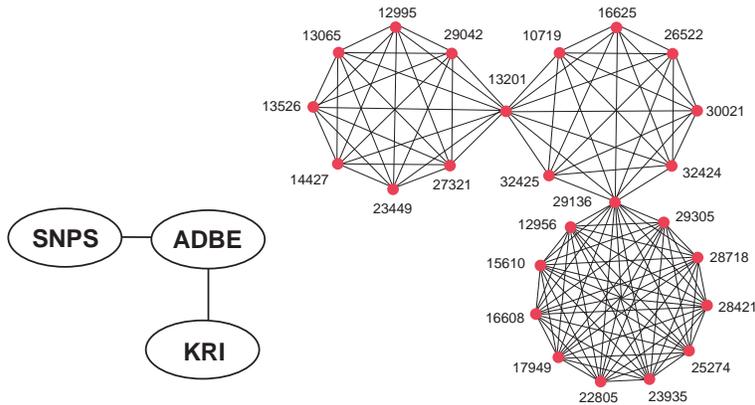


Figure 2: *The undirected graphs showing board interlocks (left) and the network of co-directorship as derived from the affiliation network illustrated in Figure 1.*

network’s edges represent membership of the former in the latter. The most natural representation of such a network, sometimes called an *affiliation* network, is a graph with two sorts of vertices—one each for boards and directors—that has edges connecting directors with the boards on which they sit. The result is an example of a *bipartite* graph: one whose vertices can be divided into two distinct sets and whose edges only make connections between the two sets. Figures 1 and 3 are examples. The board and director graphs mentioned above now appear as “projections” of the bipartite graph onto one of its two sets of vertices.

Figures 1 and 2 illustrate these issues for that part of social network of corporate governance connected to the board of Adobe Systems Inc., a software house. The former, Figure 1, shows the full bipartite graph, while Figure 2 shows the two projections. The graph appearing at the left of Figure 2 has the boards as its nodes and edges connecting interlocked boards while the graph at right has directors for nodes and includes an edge between two directors if they sit on the same board. Note that both of the projections are undirected graphs. But PageRank is, as we will see below, designed to operate on directed graphs. One of the main technical obstacles in adapting PageRank to the ranking of, for example, the boards, is to find a way to incorporate extra information into a bipartite graph such as the one in Figure 1 in such a way as to permit the construction a board projection that is a directed graph. This issue is touched on in the following section, then treated in detail in Section 4.

2.2.1 Remarks on related literature

Generally, previous analyses of corporate governance networks have not explicitly modeled boards of directors as bipartite graphs, although there are some exceptions (e.g., Newman (2003), Conyon and Muldoon (2006*b*)). Our paper (in section 3) augments such studies by considering weighted directed graphs.

The notion of ‘centrality’ of boards considered in this paper is closely related to a literature on ‘board interlocks’. A board interlock occurs when the boards of two separate organizations share a common director: namely, it is the ‘degree’ of the board. Hallock (1997) provides an econometric analysis of CEO pay determination. He demonstrates that CEO pay is higher in interlocked firms after controlling for other economic determinants. Board interlocks are routinely studied in the management literature and are often motivated by sociological considerations (e.g. Davis and Greve (1997), Pettigrew (1992), Mizruchi and Bunting (1981), Useem and Karabel (1986), Useem (1984), Haunschild (1993)). The ‘interlock’ concept is not usually analyzed as a bipartite graph as in this paper.

Recent research in accounting and finance has also stressed the importance of social networks; albeit using a different framework than that offered here. Bebchuk et al. (2007) demonstrate that CEO centrality is important for a number of firm-level outcomes. They find that CEO centrality is negatively associated with firm value, measured as industry-adjusted Tobin’s Q as well as lower industry-adjusted accounting profitability. They conclude that ‘differences in CEO centrality are an aspect of firm management and governance that deserves the attention of researchers.’ Cohen et al. (2007) focus on connections between mutual fund managers and company board members arising from shared education networks. They find that a ‘replicating portfolio of connected stocks outperforms a replicating portfolio of non-connected stocks by up to 8.4% per year’. Larcker et al. (2005) use the network of directors and firms to compute the minimum number of other company boards that are required to establish a connection between each pair of directors. It measures the strength of a communication channel between board members to influence board decisions. They find that ‘CEOs at firms where there is a relatively short back door distance between inside and outside directors or between the CEO and the members of the compensation committee earn substantially higher levels of total compensation’. Kramarz and Thesmar (2006) use French data to investigate corporate networks, showing that corporate governance is generally

worse in firms run by former civil servants. They find that ‘CEOs who are former bureaucrats are more likely to accumulate directorships, and the more they do, the less profitable is the firm they run’. Stathopoulos and Kirchmaier (2006) also find that performance is negatively correlated with director social networks. They conclude that ‘it appears that CEOs use the power developed through their social networks to the detriment of shareholders.’

In an auditing context, Iyer et al. (1997) find that alumni, defined as a company officer who previously worked for an audit firm, have significant ties with their former audit firms are more likely to provide former firms with benefits if they have stronger ties with them. Lennox and Park (2007) hypothesizes that an independent audit committee is less likely to appoint a company officer’s former audit firm. They find that ‘companies appoint officers’ former firms more often than they appoint alternative audit firms. However, companies are less likely to appoint officers’ former firms if audit committees are more independent.’ Such studies show the importance of networks in an accounting and finance corporate governance context.

Finally, we should also mention that previous corporate governance research has been very interested in constructing indices that permit one to rank outcomes. For example, Gompers et al. (2003) introduced a governance index, the ‘*G-index*’, which ranked firms based on twenty-four separate measures of the quality of corporate governance.³ The construction of such indices are potentially labor intensive. In contrast, the extension of PageRank to social networks seems a fruitful avenue to pursue since it enables one to determine, with relative ease and efficiency, the importance of a particular ‘actor’ within the whole network. For example, the ‘actor’ can be thought of as an individual, the board of directors, an auditor or indeed an entire organization. And the ranking procedure we offer here enables one to relatively quickly rank the importance of actors in the social network.

3 PageRank for boards of directors

The PageRank algorithm (Brin and Page (1998) and Brin et al. (1999)) assigns a numerical rank to each vertex in a directed graph. These ranks were originally intended as an aid to searching the World Wide Web and so have a natural interpretation in a graph whose vertices represent web pages and whose (directed) edges represent hyperlinks. PageRank then provides an assessment

³The composite ‘*G-index*’ is made-up from information about poison pills, director indemnification, golden parachutes, classified / staggered boards, anti-greenmail etc.

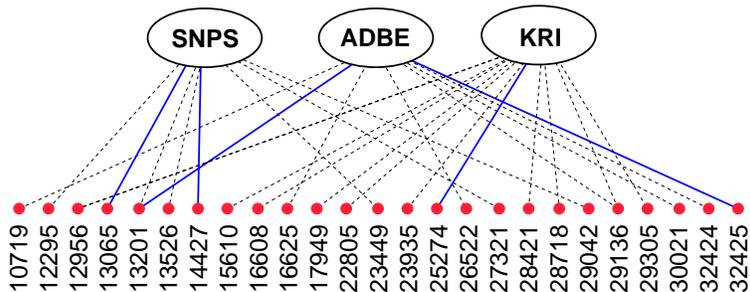


Figure 3: *Another representation of the bipartite graph in Figure 1, but now with edges weighted according to whether the director is an executive (weight is 1.0, plotted with a heavy blue edge) or a non-executive (weight is 0.0, plotted with a lighter dashed edge).*

of the importance (or authority) of a Web page (node) that is largely independent of the page’s content.

Here we describe a scheme that generalizes PageRank to the ranking of boards in the social network of corporate governance. The problem splits naturally into two pieces: deriving weighted, directed projections from the unweighted, undirected, bipartite graph that represents the affiliation network of boards of directors and (ii) computing PageRanks for the two projections.

3.1 Adding weights to a bipartite graph

The top panel of Figure 1 shows a small part of the world of the corporate governance in the US. The edges connecting boards to their directors in this graph are all the same: they are unweighted or, equivalently, all have the equal weight. More generally, one could assign a (non-negative) weight to each such edge. One might, for example, assign a Boolean weight of zero⁴ to every edge that connects a non-executive director to the board on which he or she serves and a weight of one to those edges that represent the connections between executive directors and their boards. One can represent this graphically with something like Figure 3: in Section 4 below we will introduce a scheme that processes these weights (or any others one might choose to assign) and produces a pair of weighted, directed projections.

⁴As mentioned above, this is a somewhat non-standard use of the term “weight”. In normal graph-theoretic usage an edge with zero weight is simply absent from in the graph, but here we wish to suppress the usual connection between an edge’s weight and its existence. In the bipartite graph it will be possible for edges to exist, but have zero weight.

3.2 PageRank

Here we offer a brief account of the calculations involved in generating PageRanks. For a longer discussion one might begin with the very clear and entertaining introduction by Higham and Taylor Higham and Taylor (2003), then proceed to the original references, Brin et al. (1999) and Brin and Page (1998). A more general analysis of link-based algorithms, including PageRank, appears in Wang (2004).⁵

Suppose that one has a weighted, directed graph with N vertices and an adjacency matrix W whose entries are conventional, positive weights $w_{j,k} \geq 0$ satisfying

$$\begin{aligned} w_{j,k} > 0 & \quad \text{if a (directed) edge connects vertex } k \text{ (tail) to vertex } j \text{ (tip);} \\ w_{j,k} = 0 & \quad \text{if there is no edge between } j \text{ and } k. \end{aligned}$$

Then define o_k , the *weighted out-degree* of vertex k , to be

$$o_k = \sum_{j=1}^N w_{j,k}. \tag{1}$$

That is, o_k is the sum of the weights of all the edges that reach from k to some other vertex.

The PageRank algorithm uses these quantities, as well as an adjustable parameter $0 \leq d < 1$, to define a converging sequence of ranks r_j^n . Here the subscript j ranges over the vertices in the graph (that is $1 \leq j \leq N$) while the superscript n starts from zero and counts the number of times one has applied the PageRank update rule:

$$r_j^{n+1} = (1-d)s_j + d \sum_{k=1}^N \left(\frac{w_{j,k}}{o_k} \right) r_k^n \tag{2}$$

The quantities s_j are, for reasons that will become clear shortly, sometimes called *source strengths*: they should be positive and satisfy

$$N = \sum_{j=1}^N s_j. \tag{3}$$

The sum over k appearing in (2) is best thought of as a sum over (incoming) neighbors or, in

⁵Other recent papers examining link-based algorithms and social networks include Ding et al. (2004) and Diligenti et al. (2004)

graph-theoretic terms, *predecessors*, for only those edges that start at some vertex k and end at j will contribute to the PageRank of the j -th vertex (that is, only these edges will have weights $w_{j,k} \neq 0$).

Although the iterative rule (2) tells us how to generate r_j^{n+1} given r_j^n , it cannot tell us how to start the process. By convention one chooses $r_j^0 = 1 \forall j$. That is, all the vertices start out with equal rank, then successive applications of (2) generate successive generations of ranks. It is not too hard to show that, provided $d < 1$, this iterative procedure will converge. That is, eventually it will be true that $r_j^{n+1} \approx r_j^n$. Indeed, something much stronger is true: given any small number $\delta \ll 1$, one can always choose an n_\star sufficiently large that

$$\max_j \|r_j^{n_\star+t} - r_j^{n_\star}\| \leq \delta \quad \forall t > 0.$$

In words, after n_\star iterations *all* subsequent generations of ranks will be within δ of the $r_j^{n_\star}$. In practice, we choose some small tolerance δ and repeats (2) until the first n for which $\max_j \|r_j^{n+1} - r_j^n\| \leq \delta$.

3.2.1 Interpretation as a random walk

The intuitive idea behind PageRank is clearest in the algorithm's original context, the World Wide Web. Recently, Newman (2005) has also discussed centrality measures based on random walks, namely counting how often a node is traversed by a random walk between two other nodes. In our context, imagine a deeply indecisive individual who browses the web at random. He reads a web page, then chooses a new page at random by clicking, with equal probability, on any of the links on the page he has just finished (ignore, for the moment, the possibility that his page has no hyperlinks). If such a reader persisted in his efforts he would eventually visit a very large proportion of the web's pages, most of them many times over (also ignore the fact that pages are continually being added to the web or removed from it). If he kept a list of all the pages he ever visited and also kept track of how often he visited each one then, eventually, the ratio

$$\frac{\text{Number of visits to page } j}{\text{Total number of pages visited}} \tag{4}$$

would tend to a constant, converging in a manner reminiscent of the PageRanks.

Indeed, the ratio (4) would converge precisely to r_j/N where N is the number of pages in the web and is the PageRank r_j produced by applying (2) with $d = 1$. The parameter d is, in this view, a probability: it is the probability that our random reader, having finished his page, decides to proceed as described above. As an alternative, if $d < 1$, we could permit our reader to jump, with probability $(1 - d)$, to an arbitrary new page anywhere in the web, choosing the j -th page with probability (s_j/N) . Here, N is the number of pages in the web and s_j is the source strength of the j -th page. If he pursues this mixed strategy—sometimes choosing a random link from the current page, sometimes jumping arbitrarily—then the ratio (4) will tend to r_j/N where r_j is the PageRank produced by repeated application of (2).

It is not hard to recast this view of PageRank to make it appear relevant to social networks. Suppose that instead of a random browser wandering idly through the web, we consider an item of news, information or gossip being relayed randomly along the connections of a social network. Each actor in the network, upon hearing the news, either passes it (with probability d) to a randomly chosen associate or (with probability $(1 - d)$), relays the news to some arbitrary third party (perhaps by posting a notice in some public place or writing a newspaper article). Although this is a drastically abstracted account of the propagation of information, it's not wholly implausible to imagine that it could capture some aspects of the diffusion of ideas as viewed in the large.

3.2.2 Interpretation as a weighted voting scheme

PageRank admits another interpretation suggestive of applications to social networks. If we consider a page's (or a board's) rank to be a measure of the esteem in which it is held, then the terms in the update rule (2) have natural interpretations. The source term $(1 - d)s_j$ represents a sort of natural, or intrinsic component of esteem: one may, democratically, set $s_j = 1$ for all boards or, if it seems more appropriate, assign some boards—perhaps those of particularly virtuous, innovative or profitable firms—a higher intrinsic esteem. The second term, the one involving the sum over k , is essentially a poll of the j -th node's neighbors, weighted according to both r_k^n , the esteem in which the neighboring k -th vertex is held, and the relative strength of the edge connecting vertex k to vertex j (this is the factor $w_{j,k}/o_k$, which compares the weight of the edge connecting k to j with the total strength of k 's outgoing edges). That is, praise from the praiseworthy—contributions from nodes that are themselves highly esteemed—counts more.

In this view the iteration of the PageRank update corresponds to the gradual formation of a

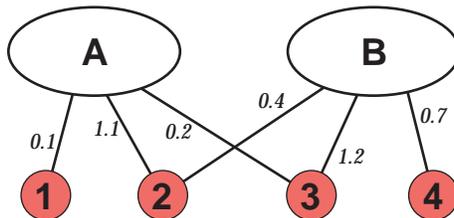


Figure 4: A weighted, undirected bipartite graph in which directors 1–4 serve on boards A and B.

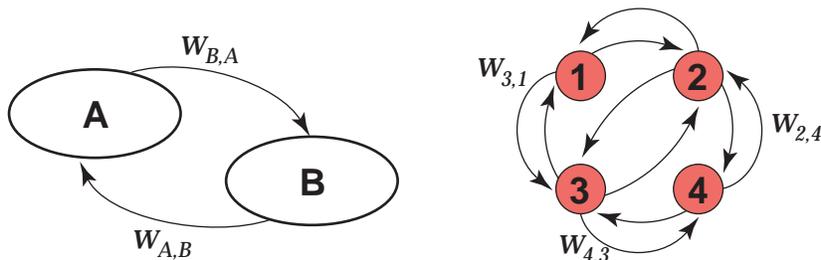


Figure 5: Directed board (left) and director (right) projections derived from the bipartite graph of Figure 4. Although all the edges in the directed graph at right have associated weights, only a few are labeled explicitly.

consensus about which nodes are most important. At the outset every node has the same rank, 1. Successive rounds of (2) redistribute esteem, treating a weighted, directed edge from k to j as a weighted vote of confidence by node k in favor of node j . Thus it is clear that the weights assigned to the edges have considerable influence on the final distribution of rank: in the next section we will describe a method for generating weights on the edges of the two projected networks from Boolean weights—the sort of Boolean weights discussed in Section 3.1—on the edges of the bipartite graph.

4 Making weighted, directed projections

Consider Figure 4, which shows a small, weighted, undirected bipartite graph. Ignoring for the moment the question of how one assigns weights to the edges, this figure is an example of the sort of data from which one might hope to derive PageRank-like measures for the boards, directors, or both. In this section we develop a method to pass from graphs like that pictured in Figure 4 to the sort of weighted, directed projections appearing in Figure 5.

4.1 Two preliminary attempts and a formula

Here we build up gradually, by way of two intermediates, to our preferred formula for the weights of edges in the bipartite graph. The main observation is that edges in the projection arise from two-edge paths in the bipartite graph. So, for example, a pair of boards A and B are connected in the board projection if they share a director. But this is the same as saying that the bipartite graph contains a pair of undirected edges, say, $E_{A,k}$ and $E_{k,B}$, where the shared director has index k . It is natural to choose the weights for the two directed edges in the projection to be a linear combination of the weights in the bipartite graph

$$\begin{aligned} w_{A,B} &= \beta W_{B,k} + (1 - \beta) W_{k,A} \\ w_{B,A} &= \beta W_{A,k} + (1 - \beta) W_{k,B} \end{aligned} \tag{5}$$

where we have introduced a new parameter $0 \leq \beta \leq 1$ that controls the relative contribution of the two weights from the bipartite graph. This is our first preliminary attempt. Notice that the two lines above are really the same formula with the roles of A and B interchanged, so one need only state one of them. Note also that the sum of the weights is conserved. That is

$$\begin{aligned} w_{A,B} + w_{B,A} &= [\beta W_{B,k} + (1 - \beta) W_{k,A}] + [\beta W_{A,k} + (1 - \beta) W_{k,B}] \\ &= [\beta W_{A,k} + (1 - \beta) W_{k,A}] + [\beta W_{B,k} + (1 - \beta) W_{k,B}] \\ &= [\beta + (1 - \beta)] W_{A,k} + [\beta + (1 - \beta)] W_{B,k} \\ &= W_{A,k} + W_{B,k} \end{aligned}$$

where, in passing from the second line to the third, we have used the fact that, as the bipartite graph is undirected, $W_{A,k} = W_{k,A}$ and $W_{B,k} = W_{k,B}$.

Of course, as Figure 4 shows, two boards may share more than one director so one might generalize (5) to

$$w_{A,B} = \sum_{k \text{ shared}} \beta W_{B,k} + (1 - \beta) W_{k,A} \tag{6}$$

where the sum runs over all shared directors: this is our second preliminary attempt. It retains the property that the sum $w_{A,B} + w_{B,A}$ is the same as the sum of all the weights on the edges that contribute to the formation of $e_{A,B}$ and $e_{B,A}$ in the projection.

Director Projection			Board Projection		
<i>Weight</i>	<i>Using (6)</i>	<i>Using (7)</i>	<i>Weight</i>	<i>Using (6)</i>	<i>Using (7)</i>
$w_{2,1}$	0.85	1.4167	$w_{A,B}$	1.375	1.8762
$w_{3,1}$	0.175	1.1667	$w_{B,A}$	1.525	2.1238
$w_{1,2}$	0.35	0.5833			
$w_{3,2}$	1.425	1.9038			
$w_{4,2}$	0.625	1.1364			
$w_{1,3}$	0.125	0.8333			
$w_{2,3}$	0.1475	2.0962			
$w_{4,3}$	0.825	0.8684			
$w_{2,4}$	0.475	0.8636			
$w_{3,4}$	1.075	1.1316			

Table 1: *Weights derived by applying (6) or (7) to the weighted bipartite graph shown in Figure 4. In both cases we used $\beta = 0.25$.*

Our second formulation, (6), is reasonably satisfactory, but in practice we prefer to make a slight modification:

$$w_{A,B} = \sum_{k \text{ shared}} \frac{\beta W_{B,k} + (1 - \beta) W_{k,A}}{(W_{B,k} + W_{k,A})/2} \quad (7)$$

That is, we scale the contribution from each pair (in the bipartite graph) by the average of its weights. This scaling means that each shared director causes the sum $w_{A,B} + w_{B,A}$ to increase by 2.

This choice of scaling arises naturally in the analysis of Boolean-weighted bipartite graphs such as the one pictured in Figure 3. The aim of the rescaling is to permit a distinction between edges that don't exist in the bipartite graph (and so don't give rise to connections in the projections) and those that do exist (and so should contribute to connections in the projection), but have zero weight. Such graphs are discussed at greater length below, but we conclude this section with Table 1, which gives sets of weights for the edges in the projections pictured in Figure 5.

4.2 Boolean weights: insiders and outsiders

Our original interest was to analyze weighted bipartite graphs such as the one illustrated in Figure 3. As we mentioned above, the weights here are somewhat unusual in that a weight of zero does not imply that the edge is absent. Rather, we imagine that the weights are Boolean variables: they reflect the answer to some “Yes”-“No” question such as “Is the director an executive director of the board to which she is connected?” One might refer to such graphs as *Boolean weighted bipartite*

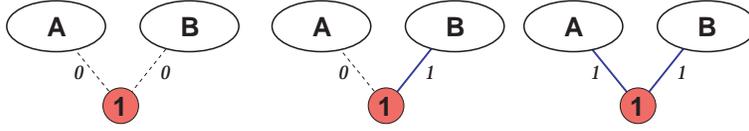


Figure 6: Three small bipartite graphs with various sets of Boolean edge weights. The weights are shown numerically and, additionally, the graphs are colored with the conventions from Figure 3.

graphs.

Figure 6 shows three of the simplest possible such graphs: each contains two boards tied together by a single shared director. To illustrate the role of the parameter β let us compute the weights in the board projection using our preferred rule (7). The leftmost graph presents an immediate difficulty since:

$$\begin{aligned}
 w_{A,B} &= \sum_{k \text{ shared}} \frac{\beta W_{B,k} + (1 - \beta) W_{k,A}}{(W_{B,k} + W_{k,A})/2} \\
 &= \frac{\beta W_{B,1} + (1 - \beta) W_{1,A}}{(W_{B,1} + W_{1,A})/2} \\
 &= \frac{\beta \times 0 + (1 - \beta) \times 0}{(0 + 0)/2} \\
 &= 0/0.
 \end{aligned}$$

One way out of this problem is to adhere to the principle that each shared director should contribute 2 to the sum $w_{A,B} + w_{B,A}$: combining this with the observation that, in this problematic case, $W_{A,1} = W_{B,1}$ we'll adopt the convention that $w_{A,B} = w_{B,A} = 1$. This has a natural generalization to the case where the two boards share several directors.

The remaining cases are easier—the formula (7) yields a sensible result without any further contemplation—and the results are summarized in Table 2. These results make qualitative sense in that if $W_{A,1} = W_{B,1}$, then $w_{A,B} = w_{B,A} = 1$. The more interesting case is when the director is an insider on only one of the two boards.

Suppose, for example, that the director is an outsider on board A, but an insider on board B. In this case the directed edges in the board projection receive different weights that depend on the parameter β . If $\beta \approx 0$ then $w_{A,B} \approx 0$ and $w_{B,A} \approx 2$, so the edge pointing from A to B is much more heavily weighted than the one running from B to A. In this case one might like to think of the heavier edge as indicating that board A is showing “esteem” for board B by recruiting

Bipartite Graph		Board Projection	
$W_{A,1}$	$W_{B,1}$	$w_{A,B}$	$w_{B,A}$
0	0	1	1
0	1	2β	$2(1 - \beta)$
1	1	1	1

Table 2: Weights derived by applying (7) to the three weighted bipartite graphs shown in Figure 6.

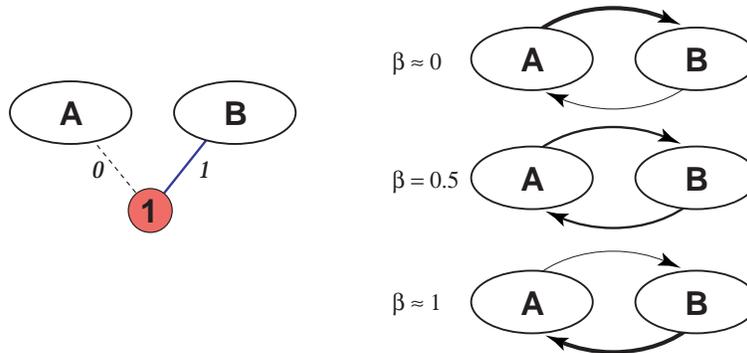


Figure 7: At left, a shared director is an insider on board B, but an outsider on board A. The three projections at right show how the parameter β influences the weights of the directed edges arising from this interlock.

one of B’s executives. Alternatively, one might think of the heavily-weighted edge as indicating “influence”: board A has a strong possibility of influencing board on B because one of A’s directors is involved in the day-to-day running of B. When $\beta = 0.5$ both edges receive the same weight: $w_{A,B} = w_{B,A} = 1$ and, finally, when $\beta \approx 1$, then $w_{A,B} \approx 2$ and $w_{B,A} \approx 0$ and the imputations about esteem and influence run in the opposite direction. The relationships between β and the weights on the directed edges are summarized in Figure 7.

5 Ranking boards of directors

5.1 The data

To implement the PageRank algorithm for the social networks of boards of directors we use two distinct data sets. One is from the United States, kindly supplied by the *Corporate Library* and the other is a data set is from the United Kingdom supplied by *Hemmington Scott* publishing. Both data sets contain an expansive list of companies and the directors who serve on their boards. The

purpose of the following empirical analysis is simply to show how the algorithm may be implemented rather than to draw any definite conclusions about the ‘true’ rank of boards which will inevitably require further analysis and comparison with other rank measures.⁶

The data for the United States are a snapshot (i.e. a cross section) of publicly traded US firms at February 2003. The data consist of 12,765 directors sitting on 1,731 boards. We report results for the largest connected component, which consists of 10,432 directors sitting on 1,452 boards. The United Kingdom (British) data are a snapshot of publicly traded UK firms at March 2002. These data describe 11,541 directors sitting on 2,236 boards. Once again we report results for the largest connected component, which here consists of 8,850 directors sitting on 1,732 boards.

5.2 Board ranks

The importance of each board (firm) is calculated using the PageRank update rule in Equation 2. We consider a Boolean weighted bipartite graph discussed in Section 3 where “yes” was the answer to “Is the director an executive director of the board to which she is connected?” We used a number proportional to the firm j ’s market capitalization as the source strength, s_j , in Equation (2). Of course, PageRanks for boards can be calculated for various values of the adjustable parameter, d : we set $d = 0.7$.⁷ We then choose values of $0 < \beta < 1$, the relative contribution of the weights in the bipartite graph. Specifically, we examined $\beta = 0.5$, $\beta = 0.1$ and $\beta = 0.9$. In Tables 3 and 4 we report the 35 highest ranked board (firms) in the United States and the United Kingdom (where $d = 0.7$ and $\beta = 0.5$).

In Table 3 General Electric, Microsoft, Exxon Mobil, Pfizer and Wal-Mart turn out to be the 5 highest ranked boards in the United States while, in Table 4, BP, GlaxoSmithKline, Vodafone Group, Lloyds TSB Group, and HSBC Holdings turn out to be the 5 top ranked boards in the United Kingdom. How are we to evaluate these results? The PageRank algorithm, applied to the board projection from a bipartite graph, gives the consensus (steady-state) solution as to which boards attract something that, in the informal motivation above, we termed “esteem”. Of course, our choices of d , β and the source strengths are illustrative. We would encourage further investigation based on alternative d and β combinations and the selection of different sources strengths (e.g.

⁶The results, therefore, are purely illustrative of a ranking technique.

⁷The choice of $d = 0.7$ may seem arbitrary but choosing $d = 0.85$, a value typically mentioned in discussions of PageRank, does not qualitatively affect the rankings reported here. For example, in the US data the correlation coefficient between the board ranks calculated separately for $d = 0.7$ and $d = 0.85$, and using $\beta = 0.5$, is 0.97.

Table 3: America's highest ranked boards. The Rank is based on the implementation of the PageRank algorithm described in the text where $d = 0.7$, source strengths are market capitalization and the β values are as indicated in the table. Data are from 2002/2003.

Company	Rank	Rank	Rank
	$\beta = 0.5$	$\beta = 0.1$	$\beta = 0.9$
General Electric Company	15.60	14.97	16.16
Microsoft Corporation	15.46	15.08	15.56
Exxon Mobil Corporation	14.50	14.77	13.99
Pfizer, Incorporated	13.98	13.89	13.92
Wal-Mart Stores, Incorporated	12.80	12.73	12.85
Citigroup, Incorporated	12.79	12.36	13.15
American International Group, Incorporated	10.58	10.23	9.94
Verizon Communications Incorporated	10.41	10.42	10.31
Johnson & Johnson	9.55	9.57	9.48
Coca-Cola Company (The)	9.14	8.81	9.47
Procter & Gamble Company (The)	9.09	9.51	8.64
International Business Machines Corporation	8.69	8.24	9.04
J.P. Morgan Chase & Co.	8.38	8.16	8.71
SBC Communications Incorporated	8.33	8.49	8.17
Bank of America Corporation	8.20	8.28	8.07
Merck & Co., Inc.	8.05	8.68	7.23
Cisco Systems, Incorporated	7.25	7.18	7.05
Fannie Mae	7.22	7.67	6.68
AOL-Time Warner, Incorporated	7.01	6.89	6.94
Viacom, Incorporated	6.65	6.29	6.84
Dell Computer Corporation	6.57	6.41	6.75
Wells Fargo & Company	6.57	6.60	6.49
ChevronTexaco Corporation	6.37	6.32	6.38
PepsiCo, Incorporated	6.28	6.20	6.35
Intel Corporation	6.26	6.38	6.13
Altria Group, Inc.	6.20	6.32	6.21
Eli Lilly & Company	5.96	6.41	5.26
Anheuser-Busch Companies	5.82	5.83	5.86
Home Depot, Inc. (The)	5.79	5.47	6.13
3M Company	5.66	5.30	5.99
Morgan Stanley	5.37	5.20	5.56
BellSouth Corporation	5.23	4.98	5.31
Amgen, Incorporated	5.20	5.47	4.92
Bristol-Myers Squibb Company	5.10	5.10	5.14
Allstate Corporation (The)	4.92	4.59	5.27

profits or employees).

But the “esteem” we measure is in a certain narrow sense a structural feature of the corporate world: it depends simply on the list of companies, the membership of their boards and their market capitalizations. In this sense our measure of esteem contrasts to other measures which rely upon the judgments of people to say whether a board is highly regarded or not, or whether the board itself promotes itself as an ideal board. Because our centrality measure does not rely on self- or other-assessment, but computes a rank based mainly on board interlocks, it is in this respect harder to manipulate and less dependent on arbitrary judgments. This is a property inherited from Page and Brin’s original PageRank, which is, by design, link-based rather than content-based.

Our rankings are in some accord with those reported in other places such as *Fortune* magazine’s annual list of America’s “Most Admired Companies”. Each year *Fortune* asks a panel of executives, directors and security analysts to rank a firm according to eight criteria: innovation, employee talent, use of corporate assets, social responsibility, quality of management, financial soundness, long-term investment value, and quality of products and services. For the “top ten” survey, respondents are asked to select the ten companies they admire most in any industry. They chose from a list of corporations that ranked in the top 25% overall last year, plus any that finished in the top 20% of their category. Six of the *Fortune* top 10 firms appear in the list in Table 3. These are Wal-Mart, General Electric, Dell Inc., Microsoft Corp., Johnson and Johnson and IBM. Also, *Fortune* produces a global “World Most Admired Companies”: all of the British companies that appear in the 2004 *Fortune* global top 50 also appear in Table 4.

Our rankings can also be compared to the “Governance Index” introduced by Gompers et al. (2003). Using data from the *Investor Responsibility Research Center*, they identify 24 distinct corporate governance provisions. These include poison pills, director indemnification, golden parachutes, classified / staggered boards, anti-greenmail etc. For each firm they add one point to the index for every provision that restricts shareholder rights, or equivalently increases managerial power. This (inverse) measure of governance quality potentially ranges from zero to twenty-four for each firm. They identify IBM, Wal-Mart, PepsiCo, American International Group as firms with a lower “G-Index”. Again, these appear in Table 3. These initial results should be taken as preliminary and we propose further tests, for example using newer or alternative data to evaluate their robustness. However, the general procedure seems to work soundly.

Table 4: Britain's highest ranked boards. The Rank is based on the implementation of the PageRank algorithm described in the text where $d = 0.7$, source strengths are market capitalization and the β values are as indicated in the table. Data are from 2002/2003.

Company	Rank	Rank	Rank
	$\beta = 0.5$	$\beta = 0.1$	$\beta = 0.9$
BP PLC	66.44	63.83	70.20
GlaxoSmithKline PLC	46.96	48.39	45.56
Vodafone Group PLC	36.16	36.49	35.19
Lloyds TSB Group PLC	33.51	33.58	30.97
HSBC Holdings PLC	33.24	34.61	31.85
Shell Transport and Trading Co PLC	25.04	26.18	24.22
AstraZeneca PLC	24.02	27.18	20.49
Royal Bank of Scotland Group (The) PLC	23.84	24.05	23.13
Rio Tinto PLC	21.38	20.12	19.67
Unilever	19.01	23.69	15.26
Diageo PLC	17.71	13.74	20.76
Barclays PLC	15.49	14.29	16.37
Reuters Group PLC	13.23	10.94	15.66
Anglo American PLC	13.16	13.82	12.61
Schroders PLC	12.72	12.77	12.41
HBOS PLC	12.57	12.14	12.53
BT Group PLC	12.53	10.64	14.67
Reckitt Benckiser PLC	12.38	12.95	10.47
Six Continents PLC	11.98	18.32	5.50
Prudential PLC	11.30	11.61	10.86
Standard Chartered PLC	10.60	7.74	11.17
Rolls-Royce PLC	10.31	14.46	7.84
Trinity Mirror PLC	10.14	11.25	9.27
BAA PLC	9.45	9.13	8.32
Johnson Matthey PLC	9.42	9.61	8.84
Boots Company (The) PLC	9.41	7.61	10.79
Legal & General Group PLC	9.18	8.92	9.38
British Airways PLC	9.07	8.52	9.19
Invensys PLC	8.87	9.50	8.30
Allied Domecq PLC	8.72	9.38	8.32
Cable and Wireless PLC	8.39	8.26	7.48
Marconi PLC	8.29	8.18	8.14
Close Brothers Group PLC	8.08	8.19	7.69
Smiths Group PLC	7.81	4.16	9.16
British Sky Broadcasting Group PLC	7.68	7.15	7.31

5.3 A simple “board rank” model

Having computed a rank for each board (vertex) in the social network, a natural question arises: What factors lead to, or are associated with, a high page rank? Here we only briefly investigate this question. To do so, we estimate a simple statistical model where the outcome variable is the rank of the j -th board (i.e. it’s “Board Rank”). We identified a set of observable firm-level variables, described below, that might be thought to influence the rank of a board.

At this stage we can only estimate the model for the USA, as the necessary data for the UK were not available to us. We supplemented the US *Corporate Library* data with a secondary firm-level data set from Standard & Poors *Execucomp* database, which tabulates many potentially useful characteristics for each firm. We used the August 2004 release, which contains company information for fiscal year 2003, such as compensation, firm size, sector, etc. After combining the two data sets, we estimated the following simple linear model for the available data:

$$(\text{Board Rank})_j = \alpha + \gamma_1 x_{1j} + \epsilon_j \quad (8)$$

where x_1 is a matrix containing the following covariates:

- (i) The degree of the vertex in the board projection. That is, the number of other boards with which a given board is interlocked.
- (ii) The size of the firm, measured as the log of total sales.
- (iii) The total compensation received by the firm’s CEO. This is measured as the sum of salary, bonus, other payments and the Black-Scholes value of options granted during the fiscal year.
- (iv) Firm performance measured as the five-year total return to shareholders (including reinvested dividends)
- (v) The proportion of outsiders on the main board
- (vi) The size of the board.
- (vii) The “Governance Index” defined by Gompers et al. (2003) and available through the *Investor Responsibility Research Center*.

Table 5: *Board Rank model: Estimation of Equation 8. The model is estimated using United States data for varying levels of β as specified. Note, + is significant at the 10% level, * significant at 5% and ** significant at 1%.*

	(1)	(2)	(3)	(4)
Variable		$\beta = 0.5$	$\beta = 0.1$	$\beta = 0.9$
Mean		Influence	Influence	Influence
Board degree	7.08	0.14** (0.01)	0.14** (0.01)	0.14** (0.01)
Log(Sales)	7.21	0.24** (0.05)	0.24** (0.05)	0.24** (0.05)
CEO pay	5.35m	0.04** (0.01)	0.04** (0.01)	0.04** (0.01)
Stock returns	7.01	0.01 (0.02)	0.01 (0.02)	0.02 (0.02)
Proportion outsiders	0.66	-0.17 (0.23)	-0.23 (0.22)	-0.11 (0.24)
Board size	9.97	0.04* (0.02)	0.04* (0.02)	0.04* (0.02)
Governance index	9.04	-0.08** (0.01)	-0.08** (0.01)	-0.08** (0.01)
Industry dummies		Yes	Yes	Yes
Observations		1263	1263	1263
R-squared		0.68	0.69	0.68

(viii) A set of 64 separate industry dummy variables. These are defined at the 2-digit standard industrial classification level.

Finally, γ_1 is the parameter to be estimated and ϵ_j is a stochastic error term. The variance covariance is made stationary (i.e. robust to arbitrary heteroscedasticity) using the method of Huber (1964) and White (1980).

The results are contained in Table 5. Column (1) provides the means of the independent variables. Column (2) to (4) contain the results from the estimation. Each model in (2) through (4) is estimated under different assumptions about the weighting parameter β . The results indicate that firms with a higher board degree centrality, greater sales, greater CEO compensation, and board size are likely to have higher recorded board ranks according to our method.

The governance index (an inverse measure of quality) is correctly signed and significant. The stock returns and the proportion of outside directors variables are insignificant. We expected these

indicators of corporate governance quality to contribute positively to board influence (rank). In separate regressions (not tabulated here) we found that re-estimating Equation 8, but excluding the board degree variable, resulted in positive and significant coefficient estimates for the proportion of outsiders. We would, therefore, encourage further modeling to build upon the preliminary results presented here. In general, our results can be seen as augmenting empirical governance research trying to understand the determination of board structure (e.g., Hermalin and Weisbach (1998), Hermalin and Weisbach (1988)).

6 Conclusions

In this paper we have proposed a novel, essentially structural metric by which the authority, importance and influence of the board can be evaluated and we have argued that our measure, which is related to the PageRank algorithm, the system at the heart of the extremely popular Google search engine, is applicable to the social network of boards of directors. Our research contributes to an emerging social network literature in corporate governance including Kramarz and Thesmar (2006), Guedj and Barnea (2007), Stathopoulos and Kirchmaier (2006), Larcker et al. (2005), Bebchuk et al. (2007), Conyon and Muldoon (2006*b*), Cohen et al. (2007), and Lennox and Park (2007).

We have made a number of contributions to the field of corporate governance. First, we have reviewed some important features of graph theory which are germane to an analysis of social networks. We began, by restating the idea that the affiliation network of the board of directors can be represented as a bipartite graph (namely two sets of vertices with edges running between unlike kinds). We illustrated in Figure 2 that the resulting projections from such a bipartite representation are undirected graphs.

We then introduced and explained the PageRank algorithm. It assigns a numerical value to each vertex in a directed graph according to the update rule given in Equation (2). We illustrated that the rank of a given vertex j depends recursively on an adjustable tuning parameter d , source strengths, s_j , and the sum over incoming neighbors for only those edges that start at some vertex k and end at j (only edges with weights $w_{j,k} \neq 0$ contribute to the rank of vertex j). We discussed two interpretations of PageRank first as a random walk and second as a weighted voting scheme. These interpretations have arisen in sister sciences, such as physics, applied mathematics and computing

science, but their application to social sciences and management research is novel.

PageRank, as we discussed, is designed to operate on directed graphs. The simple bipartite representation in Figure 1 does not capture this directionality. This raises a technical obstacle in adapting PageRank to the ranking of social networks of boards of directors. One needs to find a way to incorporate extra information into a bipartite graph (like the one in Figure 1) in such a way as to generate a board projection that is a directed graph. In Section 4 we have proposed such a method. Our paper has therefore described, uniquely, a method for calculating the PageRank algorithm for weighted directed bipartite graphs.

Finally, we implemented our code and calculated preliminary PageRank estimates of the board projections for publicly traded firms in the United States and the United Kingdom. We documented which companies can be structurally classified as “esteem-worthy” in the social network of corporate governance. In summary, we hope the procedure outlined in this paper is valuable to social network researchers investigating arbitrary affiliation networks since it permits the ready calculation of a PageRank centrality measure.

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